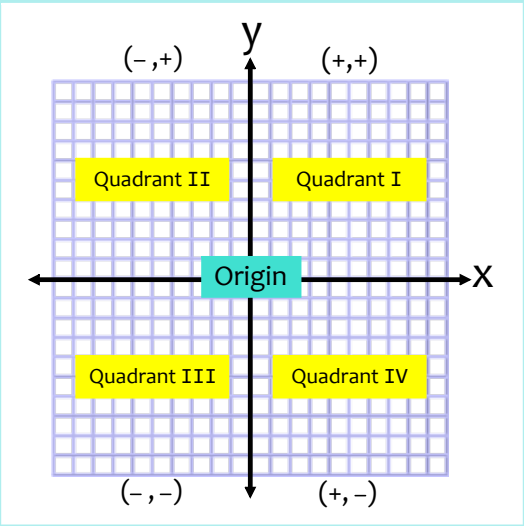
1.8 Coordinate Geometry

Coordinate Plane =

Cartesian Plane =

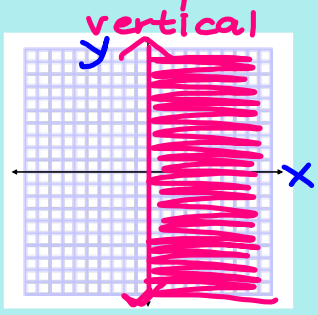
x & y axes



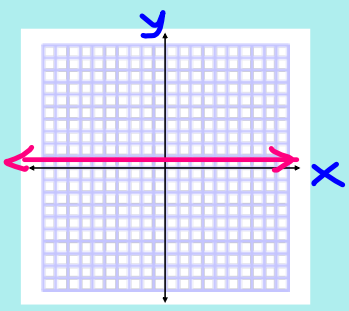
Example 1

Describe and sketch the regions given by each set.

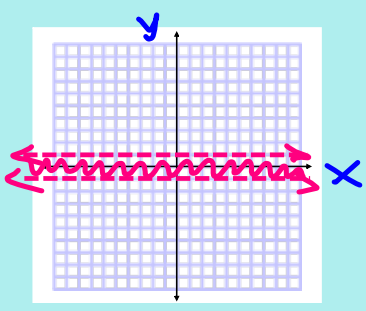
a) $\{(x,y)|x \geq 0\}$



b) $\{(x,y)|y = 1\}$



c) $\{(x,y)| |y| < 1\}$



DISTANCE FORMULA

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example 2

Which of the points P(1,-2) or Q(8,9) is closer to the point A(5,3)? P is closer to A

PA $(1,-2)$ $(5,3)$ (x_1, y_1 x_2, y_2)

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\sqrt{(5-1)^2 + (3-(-2))^2}$

$\sqrt{(4)^2 + (5)^2}$

$\sqrt{16 + 25}$

$\sqrt{41}$

QA $(8,9)$ $(5,3)$ (x_1, y_1 x_2, y_2)

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\sqrt{(5-8)^2 + (3-9)^2}$

$\sqrt{(-3)^2 + (-6)^2}$

$\sqrt{9 + 36}$

$\sqrt{45} = 3\sqrt{5}$

$3 \mid 45$
$3 \mid 15$
5

MIDPOINT FORMULA

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Example 3

Find the midpoint between P(1,-2) and Q(8,9). (x_1, y_2 x_2, y_2)

$\left(\frac{1+8}{2}, \frac{-2+9}{2}\right)$

$\left(\frac{9}{2}, \frac{7}{2}\right)$

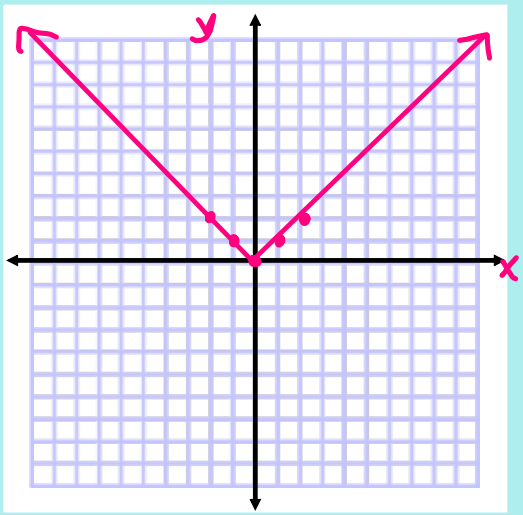
GRAPHS OF EQUATIONS

Example 4

Sketch the graph of y = |x| (absolute value).

x	y
-2	2
-1	1
0	0
1	1
2	2

V – shape



The points where the graph crosses the axes are called intercepts.

To find the x-intercept, plug in a 0 for y & solve.

To find the y-intercept, plug in a 0 for x & solve.

Example 5

Find the x- and y-intercepts of the graph of the equation $y = x^2 - 2$.

x-int

$0 = x^2 - 2$

$+2 \quad +2$

$\sqrt{2} = \sqrt{x^2}$

$\pm\sqrt{2} = x$

$(\sqrt{2}, 0)$ & $(-\sqrt{2}, 0)$

y-int

$y = 0^2 - 2$

$y = -2$

$(0, -2)$

Example 5 is a a quadratic equation. The graphs of quadratics are U-shaped and have a vertex. The x-value of the vertex is always in the middle of the x-values of the x-intercepts.

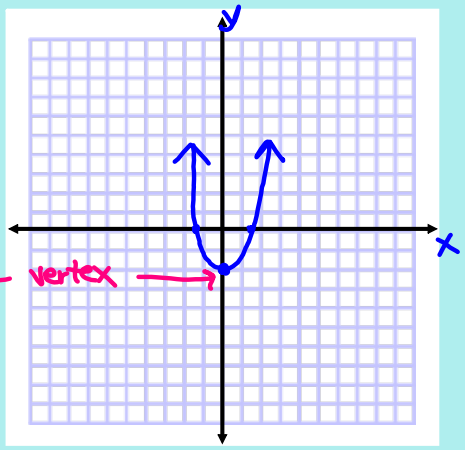
Example 6

Graph $y = x^2 - 2$ using the intercepts & the vertex.

x-int: $(\pm\sqrt{2}, 0)$

y-int: $(0, -2)$

also vertex



CIRCLES

Standard Equation of a Circle

$$(x - h)^2 + (y - k)^2 = r^2$$

Center (h,k) opp. of what you see

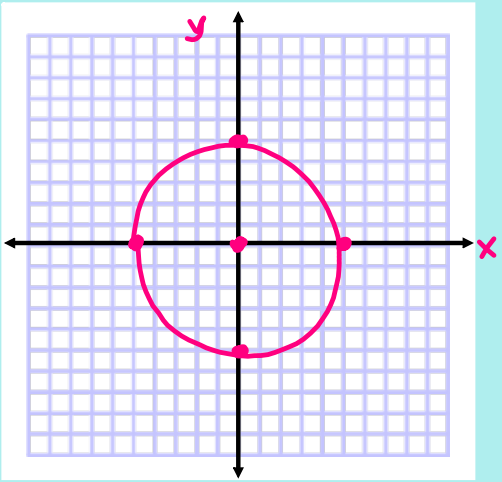
Radius = r

Example 7

Graph $x^2 + y^2 = 25$. $r = 5$

$h = 0$ $k = 0$

center $(0,0)$



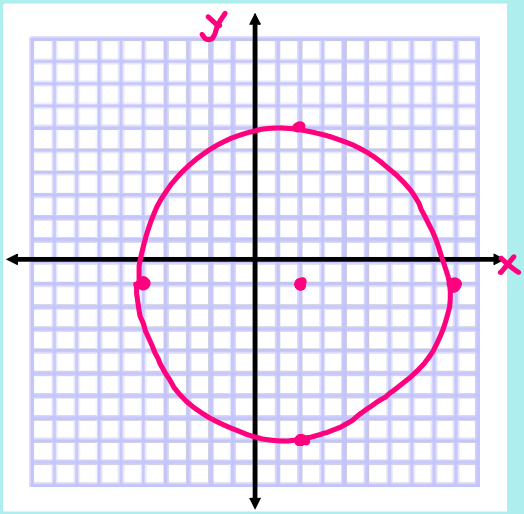
Example 8

Graph $(x - 2)^2 + (y + 1)^2 = 49$.

$r = 7$

$h = 2$ $k = -1$

center $(2, -1)$



Example 9

Find an equation of a circle with radius 3 and center $(2, -5)$.

h k

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-2)^2 + (y+5)^2 = 9$$

Example 10

$(x-h)^2 + (y-k)^2 = r^2$

Find the center and radius of

$x^2 + y^2 + 2x - 6y + 7 = 0.$

$x^2 + 2x \underline{+1} + y^2 - 6y \underline{+9} = -7 \underline{+1} \underline{+9}$

$\frac{1}{2}(2) = 1 \quad (1)^2 = 1$

$\frac{1}{2}(-6) = -3 \quad (-3)^2 = 9$

$(x+1)^2 + (y-3)^2 = 3$

center $(-1, 3)$

$r = \sqrt{3}$

Example 11

Find the center and radius of

$x^2 + y^2 - 2x + 4y + 1 = 0.$

$x^2 - 2x \underline{+1} + y^2 + 4y \underline{+4} = -1 \underline{+1} \underline{+4}$

$\frac{1}{2}(-2) = -1 \quad (-1)^2 = 1$

$\frac{1}{2}(4) = 2 \quad (2)^2 = 4$

$(x-1)^2 + (y+2)^2 = 4$

center $(1, -2)$

$r = 2$

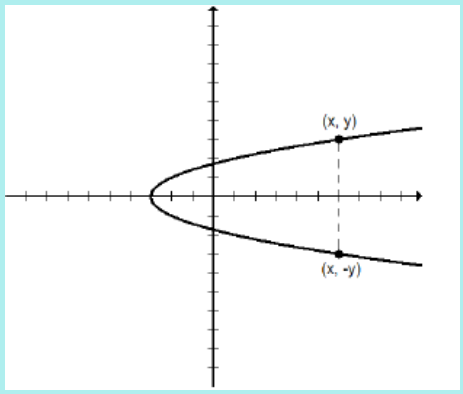
Symmetry

$x = (-y)^2 - 3$
$x = y^2 - 3$ Same
$x = y^2 - 3$

Symmetry in the x-axis

Meaning: (x,-y) is on the graph whenever (x,y) is

Testing: Plug in -y for y.
Do you get an equivalent equation?



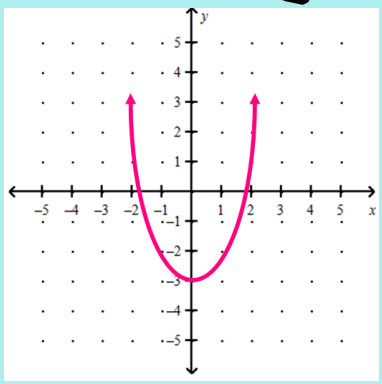
Symmetry

$y = (-x)^2 - 3$
$y = x^2 - 3$ Same
$y = x^2 - 3$

Symmetry in the y-axis

Meaning: (-x,y) is on the graph whenever (x,y) is

Testing: Plug in -x for x.
Do you get an equivalent equation?



Symmetry

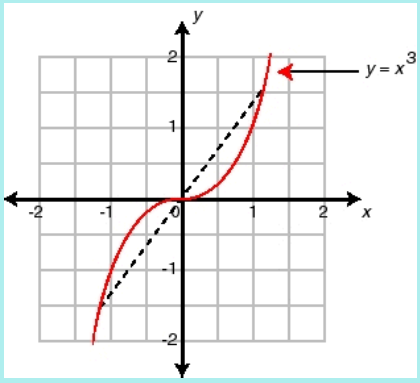
$-y = (-x)^3$
$-y = -x^3$
$y = x^3$ Same

$y = x^3$

Symmetry in the origin

Meaning: (-x,-y) is on the graph whenever (x,y) is

Testing: Plug in -x for x and -y for y. Do you get an equivalent equation?



Example 12

Test the equation for symmetry and sketch its graph.

$y = x^3 - 9x$

x	y
0	$0^3 - 9(0) = 0$
1	$1^3 - 9(1) = -8$
2	$2^3 - 9(2) = -10$
3	$3^3 - 9(3) = 0$

y-axis sym.

$y = (-x)^3 - 9(-x)$

✗ $y = -x^3 + 9x$

x-axis sym.

$\frac{-y}{-1} = \frac{x^3}{-1} - \frac{9x}{-1}$

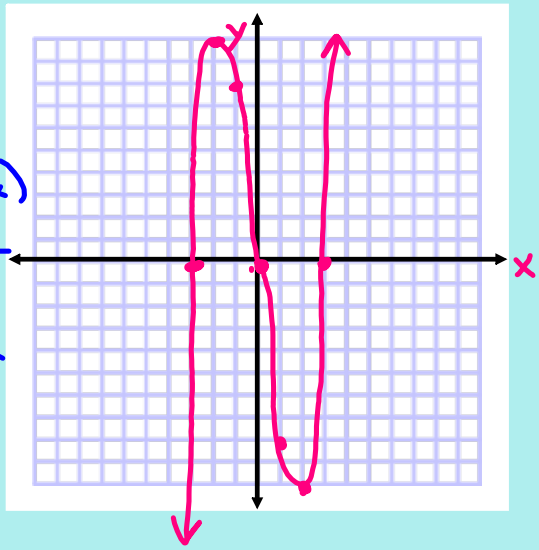
✗ $y = -x^3 + 9x$

origin sym.

$-y = (-x)^3 - 9(-x)$

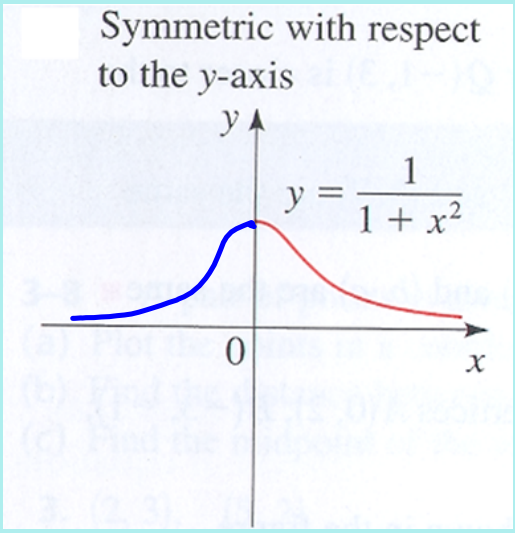
$\frac{-y}{-1} = \frac{-x^3}{-1} + \frac{9x}{-1}$

✓ $y = x^3 - 9x$



Example 13

Complete the graph using the given symmetry property.



Example 14

Complete the graph using the given symmetry property.