2.2 SLOPE AND RATE OF CHANGE

DEFINITION: $\text{slope} = \frac{\text{rise}}{\text{run}}$

FORMULA: $m = \frac{y_2 - y_1}{x_2 - x_1}$

ALWAYS SIMPLIFY FRACTIONS!

a) $\frac{10}{4} \div 2 \div 2$ $\frac{5}{2}$

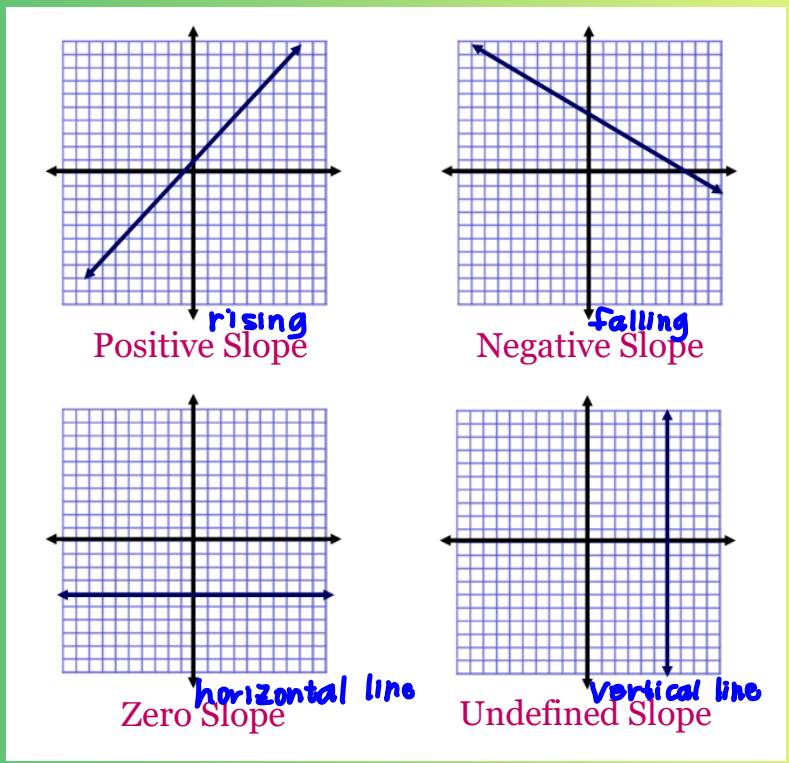
b) $\frac{3}{0}$ undefined

c) $\frac{0}{4}$ 0

d) $\frac{-6}{-5}$ $\frac{6}{5}$

e) $\frac{1.25}{3}$ $\frac{5}{12}$

CLASSIFICATION OF LINES BY SLOPE



Examples: Find the slope of the line passing through the given points. Then tell whether the line rises, falls, is horizontal, or is vertical.

1. $(-3,5)$ & $(2,1)$ (x_1, y_1) (x_2, y_2)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-5}{2+3} = \frac{-4}{5} = m \rightarrow \text{falling}$$

2. $(-2,-4)$ & $(-2,-1)$ (x_1, y_1) (x_2, y_2)

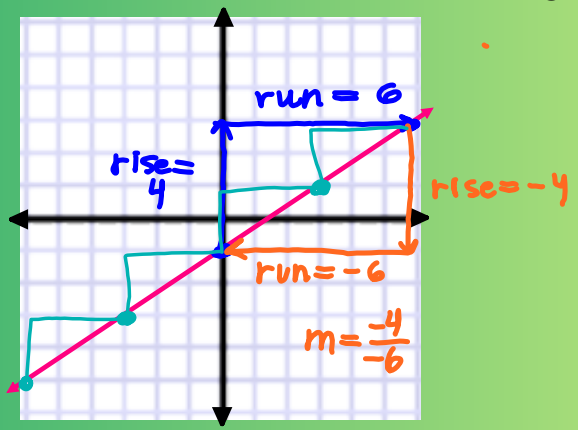
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1+4}{-2+2} = \frac{3}{0} \rightarrow m = \text{undefined}, \text{vertical}$$

3. $(-6,-1)$, $(0,9)$ (x_1, y_1) (x_2, y_2)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9+1}{0+6} = \frac{10 \div 2}{6 \div 2} \rightarrow m = \frac{5}{3}, \text{rising}$$

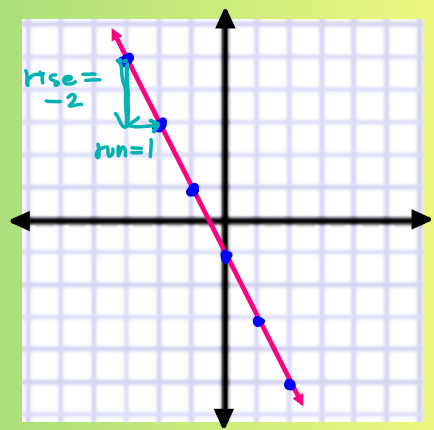
Examples: Find the slope of each line graphed below.

4.



$$m = \frac{\text{rise}}{\text{run}} = \frac{4 \div 2}{6 \div 2} = \frac{2}{3}$$

5.



$$m = \frac{-2}{1} = -2$$

Two lines in a plane are **parallel** if they do not intersect. The slopes of parallel lines are _____the same_____.

Two lines in a plane are **perpendicular** if they intersect to form a right angle. The slopes of perpendicular lines are _____opposite reciprocals_____.

w/n → 1

n/w

opposite → diff. signs

reciprocals → flip

Examples

6. If $m = \frac{2}{5}$, then the m of the $\perp$ line is $-\frac{5}{2}$.

perpendicular → $\perp$

7. If $m = -3$, then the m of the $\perp$ line is $\frac{1}{3}$.

$-3 = \frac{-3}{1}$

8. Tell whether the lines through the following points are parallel, perpendicular, or neither. Also, tell which line is steeper.

Line 1: (-3,3) and (3,-1) (x_1, y_1) (x_2, y_2)

Line 2: (-2,-3) and (2,3) (x_1, y_1) (x_2, y_2)

Line 1: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-3}{3+3} = \frac{-4 \div 2}{6 \div 2} = -\frac{2}{3}$

Line 2: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3+3}{2+2} = \frac{6 \div 2}{4 \div 2} = \frac{3}{2}$

opp. rec.

Line 1 $\perp$ Line 2

Line 2 is steeper

Examples

9. Tell whether the lines through the following points are parallel, perpendicular, or neither. Also, tell which line is steeper.

Line 1: $(-3,1)$ and $(3,4)$

Line 2: $(-4,-3)$ and $(4,1)$

Line 1: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-1}{3-3} = \frac{3 \div 3}{6 \div 3} = \frac{1}{2}$

Line 2: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-1-3}{4+4} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}$

} same

Line 1 // Line 2

neither is steeper

Examples

10. Tell whether the lines through the following points are parallel, perpendicular, or neither. Also, tell which line is steeper.

Line 1: $(\frac{1}{2}, -\frac{15}{8})$ and $(-4,-3)$

Line 2: $(8,6)$ and $(-12,1)$

Line 1: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 + \frac{15}{8}}{-4 - \frac{1}{2}} = \frac{-\frac{9}{8}}{-\frac{9}{2}} = \frac{1}{4}$

Line 2: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-6}{-12-8} = \frac{-5}{-20} = \frac{1}{4}$

} same

Line 1 // Line 2

neither is steeper