


1.7 INEQUALITIES


Inequalities generally have infinitely many solutions, which form an interval(s).

Remember when solving inequalities, if you multiply or divide **BOTH** sides by a negative number, you must **FLIP** the inequality sign.


EXAMPLE 1 Solve and sketch the solution set.

$$\begin{array}{r}
 3x < 9x + 4 \\
 \underline{-9x \quad -9x} \\
 -6x < 4 \\
 \underline{-6 \quad -6} \\
 x > -\frac{2}{3}
 \end{array}$$



EXAMPLE 2 Solve and sketch the solution set.

$$\begin{array}{r}
 6 - x \geq 2x + 9 \\
 \underline{+x \quad +x} \\
 6 \geq 3x + 9 \\
 \underline{-9 \quad -9} \\
 -3 \geq 3x \\
 \underline{3 \quad 3} \\
 -1 \geq x \\
 x \leq -1
 \end{array}$$


EXAMPLE 3 Solve and sketch the solution set of the compound inequality.

$$\begin{array}{r} 4 \leq 3x - 2 \leq 13 \\ \hline +2 \quad +2 \quad +2 \\ \frac{6}{3} \leq \frac{3x}{3} \leq \frac{15}{3} \\ \hline 2 \leq x \leq 5 \end{array}$$


EXAMPLE 4 Solve and sketch the solution set of the compound inequality.

$$\begin{array}{r} -1 < 2x - 5 < 7 \\ \hline +5 \quad +5 \quad +5 \\ \frac{4}{2} < \frac{2x}{2} < \frac{12}{2} \\ \hline 2 < x < 6 \end{array}$$


STEPS FOR SOLVING NON-LINEAR INEQUALITIES

1. Move all terms to one side.
2. Factor (complete the square or use the quadratic formula).
3. Find the values for which each factor is zero.
4. Make a number line to test the signs (positive/negative).
5. Solve. Be sure to check whether the inequality is satisfied by some or all of the endpoints of the intervals.

QUADRATIC INEQUALITIES

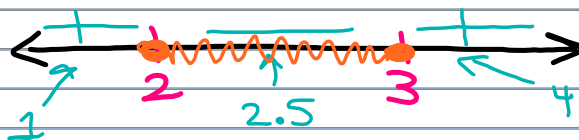
EXAMPLE 5 Solve and sketch the solution set.

$$x^2 - 5x + 6 \leq 0$$

$$(x-2)(x-3) \leq 0 \quad \text{want negatives}$$

$$x-2=0 \quad x-3=0$$

$$x=2 \quad x=3$$



$$\begin{array}{l} (1-2)(1-3) \\ (-1)(-2) \\ 2 \end{array}$$

$$\begin{array}{l} (2.5-2)(2.5-3) \\ (.5)(-.5) \\ -.25 \end{array}$$

$$\begin{array}{l} (4-2)(4-3) \\ (2)(1) \\ 2 \end{array}$$

$$2 \leq x \leq 3 \\ [2, 3]$$

QUADRATIC INEQUALITIES

EXAMPLE 6 Solve and sketch the solution set.

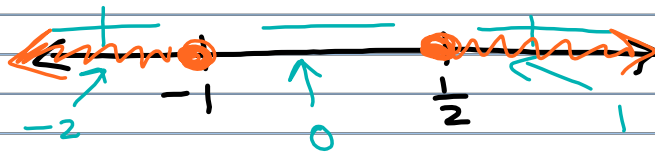
$$2x^2 + x \geq 1$$

$$\frac{-1}{2} \quad \frac{2}{2} \quad \frac{1}{2} \quad \frac{-1}{2} \quad \frac{-1}{2}$$

$$\frac{2x^2 + x - 1 \geq 0}{(2x-1)(x+1) \geq 0} \text{ positives}$$

$$2x-1=0 \quad x+1=0$$

$$x=\frac{1}{2} \quad x=-1$$



$$(-4-1)(-2+1) \quad (0-1)(0+1) \quad (2-1)(1+1)$$

$$(-\infty, -1] \cup [\frac{1}{2}, \infty)$$

INEQUALITIES WITH QUOTIENTS

1. Set equal to zero.
2. Get a common denominator.
3. Set numerator and denominator equal to zero.
4. Use a number line to find correct interval(s).
5. Don't forget to check the endpoints!

EXAMPLE 7 Solve and sketch the solution set.

$$\frac{1+x}{1-x} \geq 1$$

$$\frac{1+x}{1-x} - 1 \geq 0$$

$$\frac{1+x}{1-x} + \frac{-1+x}{1-x} \geq 0$$

$$\frac{2x}{1-x} \geq 0$$

$$2x=0 \quad x=0 \checkmark$$

$$1-x=0 \quad x=1 \times$$

$$\frac{2(-1)}{1-(-1)} \quad \frac{2(\frac{1}{2})}{1-\frac{1}{2}} \quad \frac{2(2)}{1-2}$$

$$\frac{-2}{2} \quad \frac{1}{\frac{1}{2}} \quad \frac{-4}{-1}$$

INEQUALITIES WITH QUOTIENTS

EXAMPLE 8 Solve and sketch the solution set.

$$x < \frac{2}{x-1}$$

$$x - \frac{2}{x-1} < 0$$

$$\frac{x(x-1) - 2}{x-1} < 0$$

$x+1=0$
 $x=-1$

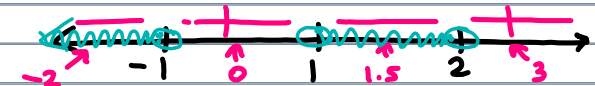
$$\frac{x^2-x}{x-1} - \frac{2}{x-1} < 0$$

$x-2=0$
 $x=2$

$$\frac{x^2-x-2}{x-1} < 0$$

$x-1=0$
 $x=1$

$$\frac{(x+1)(x-2)}{x-1} < 0$$



$(x+1)(x-2)$ at $x=-1$ and $x=2$
 $(x+1)(x-2)$ at $x=0$ and $x=1$
 $(x+1)(x-2)$ at $x=1.5$ and $x=1$

$(-\infty, -1) \cup (1, 2)$

~~$(3, \infty)$~~

ABSOLUTE VALUE INEQUALITIES

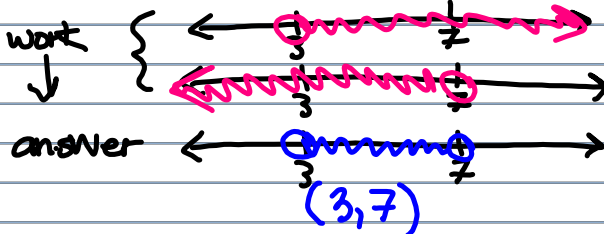
EXAMPLE 9 Solve and sketch the solution set.

$|x-5| < 2$ & what they have in common / overlap

$$\begin{array}{r} x-5 > -2 \\ +5 \quad +5 \\ \hline x > 3 \end{array}$$

and

$$\begin{array}{r} x-5 < 2 \\ +5 \quad +5 \\ \hline x < 7 \end{array}$$



ABSOLUTE VALUE INEQUALITIES

EXAMPLE 10 Solve and sketch the solution set.

$$|3x + 2| \geq 4$$

$$\begin{array}{l} 3x + 2 \leq -4 \quad \text{OR} \quad 3x + 2 \geq 4 \\ \frac{3x + 2}{-2} \leq \frac{-4}{-2} \quad \text{OR} \quad \frac{3x + 2}{-2} \geq \frac{4}{-2} \\ \frac{3x}{3} \leq \frac{-6}{3} \quad \text{OR} \quad \frac{3x}{3} \geq \frac{2}{3} \\ x \leq -2 \quad \text{OR} \quad x \geq \frac{2}{3} \end{array}$$

merge/
combine

