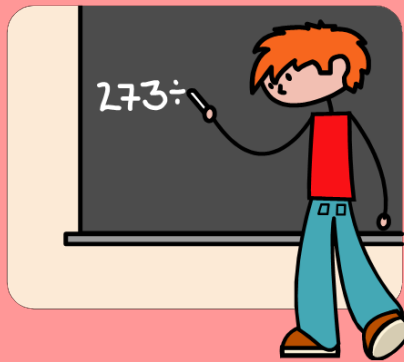


2.5 REASONING WITH PROPERTIES FROM ALGEBRA



ALGEBRAIC PROPERTIES OF EQUALITY

Addition Property	if $a = b$, then $a + c = b + c$
Subtraction Property	if $a = b$, then $a - c = b - c$
Multiplication Property	if $a = b$, then $a \cdot c = b \cdot c$
Division Property	if $a = b$, then $\frac{a}{c} = \frac{b}{c}$
Substitution Property	if $a = b$, then a may be replaced by b in any equation
Distributive Property	$a(b + c) = ab + ac$

MORE PROPERTIES

Reflexive Property	For any real #, $a = a$.
	For any segment, $AB = AB$.
	For any angle, $m\angle A = m\angle A$.
Symmetric Property	If $a = b$, then $b = a$.
	If $AB = CD$, then $CD = AB$.
	If $m\angle A = m\angle B$, then $m\angle B = m\angle A$.
Transitive Property	If $a = b$ & $b = c$, then $a = c$.
	If $AB = BC$ & $BC = CD$, then $AB = CD$.
	If $m\angle A = m\angle B$ & $m\angle B = m\angle C$, then $m\angle A = m\angle C$.

Example 1

Name the property of equality that justifies each statement.

STATEMENTS	REASONS
a. If $AB + BC = DE + BC$, then $AB = DE$.	a. subtraction prop.
b. $m\angle ABC = m\angle ABC$	b. reflexive property
c. If $XY = PQ$ & $XY = RS$, then $PQ = RS$.	c. substitution property
d. If $\frac{1}{3}x = 5$, then $x = 15$.	d. multiplication property
e. If $2x = 9$, then $x = \frac{9}{2}$.	e. division property

Example 2

Name the property of equality that justifies each statement.

STATEMENTS	REASONS
a. If $\frac{3x}{3} = \frac{120}{3}$, then $x = 40$.	a. division property
b. If $12 = AB$, then $AB = 12$.	b. symmetric property
c. If $AB = BC$ & $BC = CD$, then $AB = CD$.	c. transitive property
d. If $y = 75$ & $y = m\angle A$, then $m\angle A = 75$.	d. substitution property

Example 3Justify each step in solving $\frac{3x+5}{2} = 7$.

STATEMENTS	REASONS
1. $\frac{3x+5}{2} = 7 \cdot 2$	1. given
2. $2\left(\frac{3x+5}{2}\right) = 2(7)$	2. multiplication prop.
3. $3x + 5 = 14$ $-5 \quad -5$	3. mult. prop.
4. $3x = 9$	4. subtraction prop.
5. $x = 3$	5. division prop.

Example 3 is a proof of the conditional

$$\text{If } \frac{3x+5}{2} = 7, \text{ then } x = 3.$$

The given information comes from the hypothesis of the conditional.

It is the starting point of the proof.

The conclusion, $x = 3$, is the end of the proof.

Reasons (properties) listed for each step leading to the conclusion make this sequence a proof.

This type of proof is called a two-column proof.

Example 4

Justify each step in solving $\frac{x}{3} + 4 = 1$.

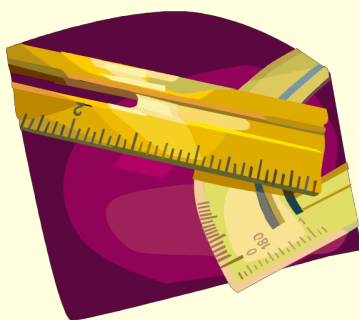
STATEMENTS	REASONS
1. $\frac{x}{3} + 4 = 1$ $\quad \quad -4 \quad -4$	1. given
2. $\frac{x}{3} = -3 \cdot 3$	2. subtraction prop.
3. $x = -9$	3. mult. prop.

Example 5

Solve the equation $4x + 9 = -3x + 2$ and write a reason for each step.

STATEMENTS	REASONS
1. $4x + 9 = -3x + 2$ $\quad \quad \quad \underline{+3x} \quad \quad \quad \underline{+3x}$	1. given
2. $7x + 9 = 2$ $\quad \quad \quad \underline{-9} \quad \quad \quad \underline{-9}$	2. addition property
3. $\frac{7x}{7} = \frac{-7}{7}$	3. subtraction property
4. $x = -1$	4. division property

Proofs in geometry can be organized in the same manner as the past three examples.



Algebra properties, definitions, postulates, and previously-proven theorems can be used for reasons.

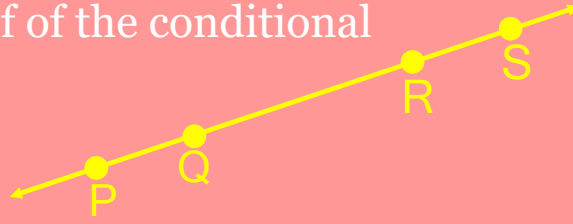
Proofs in geometry are usually written in two-column or in paragraph form.

Example 6

Justify the steps for the proof of the conditional

*If $PR = QS$, then $PQ = RS$.*Given: $PR = QS$ Prove: $PQ = RS$

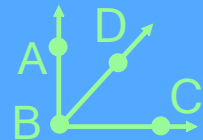
Proof:



STATEMENTS	REASONS
1. $PR = QS$	1. given
2. $PQ + QR = PR$ $QR + RS = QS$	2. seg. add. post.
3. $PQ + QR = QR + RS$ $-QR \quad -QR$	3. substitution prop.
4. $PQ = RS$	4. subtraction prop.

Example 6

Justify the steps for the proof of the conditional

*If $\angle ABD$ and $\angle DBC$ are complementary, then $\angle ABC$ is a right angle.*Given: $\angle ABD$ and $\angle DBC$ are complementary.Prove: $\angle ABC$ is a right angle.

STATEMENTS	REASONS
1. $\angle ABD$ & $\angle DBC$ are complementary	1. given
2. $m\angle ABD + m\angle DBC = 90$	2. def. of comp. \angle s
3. $m\angle ABD + m\angle DBC = m\angle ABC$	3. \angle add. post.
4. $m\angle ABC = 90$	4. substitution prop.
5. $\angle ABC$ is a right angle.	5. def. of right \angle