2.2
ANALYZE CONDITIONAL STATEMENTS

CONDITIONAL STATEMENT

- has 2 parts ⟶ **hypothesis** & **conclusion**
- written in ***if-then*** form
- hypothesis ⟶ ***if*** part
- conclusion ⟶ ***then*** part

For example...

If it is raining, then the flowers are watered.

hypothesis conclusion

Example 1

Rewrite the conditional statement in if-then form.

subject

a) ***All sharks have a boneless skeleton.***

If an animal is a shark,
then it has a boneless skeleton.

b) ***Complementary angles have a sum of 90.***

If two angles are complementary,
then they have a sum of 90.

c) ***Three points are collinear if there is a line containing them.***

If there is a line containing 3 points,
then they are collinear.

The negation of a statement is the *opposite* of the original statement.

For example...

Statement: ***The house is white.***

Negation: ***The house is not white.***

Statement: ***The puppy is not hyper.***

Negation: The puppy is hyper.

Conditional statements can be true or false.

**To show that it is *true*,
you must prove that the conclusion is
true EVERY TIME the hypothesis is true.**

**To show that it is *false*,
you need to give ONLY ONE counterexample.**

For example...

If $x^2 = 16$, then $x = 4$.

The counterexample is $x = -4$.

Example 2

Write a counterexample to show the following conditional is false.

a) *If a number is odd, then it is divisible by 3.*

true

trying to disprove

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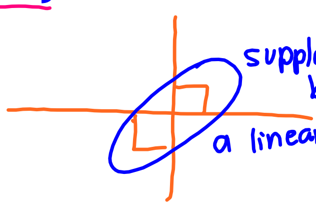
b) *Angles that are supplementary must make a linear pair.*

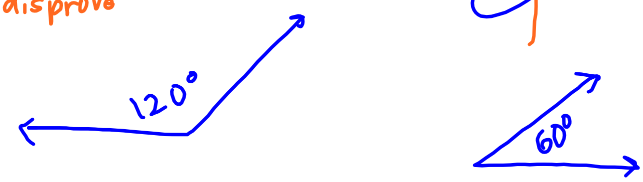
true

If angles are supplementary, then they make a linear pair.

trying to disprove



supplementary but not a linear pair



FLIP IT

The converse of a conditional statement occurs by *switching* the hypothesis and the conclusion.

For example...

Statement: ***If you see lightning, then you hear thunder.***

Converse: ***If you hear thunder, then you see lightning.***

NOT IT

The inverse of a conditional statement occurs by *negating* the hypothesis and the conclusion.

For example...

Statement: ***If you see lightning, then you hear thunder.***

Inverse: ***If you don't see lightning, then you don't hear thunder.***

FLIP IT & NOT IT

The **contrapositive** of a conditional statement occurs by *switching and negating* the hypothesis and the conclusion.

For example...

Statement: *If you see lightning, then you hear thunder.*

Contrapositive: *If you don't hear thunder, then you don't see lightning.*

ALL TOGETHER

Original	If $m\angle A = 30^\circ$, then $\angle A$ is acute.
Inverse NOT IT	If $m\angle A \neq 30^\circ$, then $\angle A$ is not acute.
Converse FLIP IT	If $\angle A$ is acute, then $m\angle A = 30^\circ$.
Contrapositive FLIP IT & NOT IT	If $\angle A$ is not acute, then $m\angle A \neq 30^\circ$.

Example 3

For the conditional statement write the

a) converse FLIP IT
b) inverse NOT IT
c) contrapositive FLIP IT & NOT IT

If an animal is a fish, then it can swim.

a) If an animal can swim, then it is a fish.
b) If an animal is not a fish, then it cannot swim.
c) If an animal can't swim, then it's not a fish.

Example 4

For the conditional statement write the

a) converse FLIP IT
b) inverse NOT IT
c) contrapositive FLIP IT & NOT IT

If angles are adjacent, they have a common side.

Adjacent angles have a common side.

a) If angles have a common side, then they are adjacent.
b) If angles are not adjacent, then they do not have a common side.
c) If angles don't have a common side, then they are not adjacent.

Example 5

For the conditional statement write the

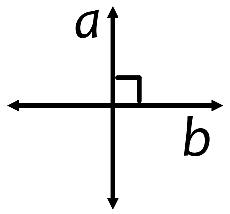
a) converse
b) inverse
c) contrapositive

If an angle measures 84°, then it is acute. true

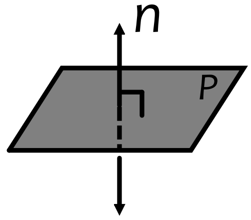
An angle that measures 84° is acute.

a) If an angle is acute, then it measures 84°. false
b) If an angle doesn't measure 84°, then it isn't acute. false
c) If an angle is not acute, then it doesn't measure 84°. true

Two lines are *perpendicular* lines if they intersect to form a right angle.



$a \perp b$

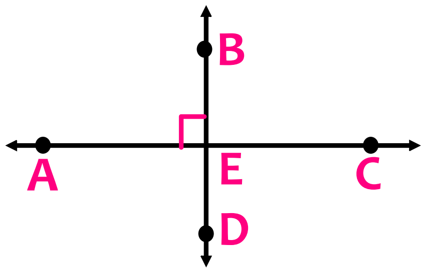


$n \perp P$

A *line perpendicular to a plane* is a line that intersects the plane in a point and is perpendicular to every line in the plane that intersects it.

Example 6

Determine whether each statement about the diagram is true. Explain your answer using the definitions you have learned.



a) $\overleftrightarrow{AC} \perp \overleftrightarrow{BD}$

true $\overleftrightarrow{AC}$ & $\overleftrightarrow{BD}$ intersect to form a right $\angle$

b) $\angle AEB$ and $\angle CEB$ are a linear pair.

true $\angle AEB$ & $\angle CEB$ are adjacent & form a line

c) $\overrightarrow{EA}$ and $\overrightarrow{EB}$ are opposite rays.

false $\overrightarrow{EA}$ & $\overrightarrow{EB}$ don't make a line

A conditional statement and its contrapositive are either *both true* or *both false*.

The converse and inverse are also either *both true* or *both false*.

Two statements that are both true or both false are called *equivalent statements*.

When a conditional statement and its converse are *both true*,

you can write them as a *single* biconditional statement...

which means it contains the phrase "if and only if."

IFF

For example...

If two lines intersect to form a right angle, then they are perpendicular. true

If two lines are perpendicular, then they intersect to form a right angle. true

Two lines are perpendicular if and only if they intersect to form a right angle.

Example 7

Rewrite the biconditional statement as a conditional statement and its converse.

original

Three lines are coplanar if and only if they lie in the same plane.

If three lines are coplanar, then they lie in the same plane.

If three lines lie in the same plane, then they are coplanar.

Example 8

Rewrite as a biconditional statement.

An angle whose measure is 180° is a straight angle.

If an angle measures 180°, then it is a straight angle. orig.

If an angle is straight, then it measures 180°. conv.

An angle measures 180° IFF it is a straight angle.