### 1.6 Angle Pair Relationships

#### Special Types of Angles

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<th>Special Types of Angles</th>
<th>Definition</th>
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<td>adjacent angles</td>
<td>angles in the same plane that have a common vertex and a common side, but no common interior points</td>
<td>( \angle 3 , \angle 4 ), ( \angle 1 , \angle 2 )</td>
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<tr>
<td>vertical angles</td>
<td>two nonadjacent angles formed by two intersecting lines</td>
<td>( \angle 2 , \angle 4 ), ( \angle 1 , \angle 3 )</td>
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<tr>
<td>linear pair</td>
<td>adjacent angles whose noncommon sides are opposite rays</td>
<td>( \angle 1 , \angle 2 ), ( \angle 3 , \angle 4 ), ( \angle 2 , \angle 3 ), ( \angle 1 , \angle 4 )</td>
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**Example 1**

a) Identify all linear pairs in the figure.

\[ \angle 1 \text{ } \& \text{ } \angle 5 \]
\[ \angle 5 \text{ } \& \text{ } \angle 4 \]

b) Identify all pairs of vertical angles in the figure.

\[ \angle 1 \text{ } \& \text{ } \angle 4 \]

**Example 2**

a) Identify all linear pairs in the figure.

\[ \angle 1 \text{ } \& \text{ } \angle 2 \]
\[ \angle 2 \text{ } \& \text{ } \angle 3 \]

b) Identify all pairs of vertical angles in the figure.

\[ \angle 1 \text{ } \& \text{ } \angle 3 \]
Vertical angles are **congruent**.

The sum of the measures of the angles in a linear pair is 180.

**Example 3**
Use the figure to the right to fill in the blanks.

a) If \( \angle 2 = 40^\circ \), then \( \angle 4 = \) \( 40^\circ \).

b) If \( \angle 1 = 105^\circ \), then \( \angle 2 = \) \( 75^\circ \).

c) If \( \angle 3 = 97^\circ \), then \( \angle 1 = \) \( 97^\circ \).

d) If \( \angle 4 = 62^\circ \), then \( \angle 3 = \) \( 118^\circ \).

**Example 4**
In the figure, \( \overline{GH} \) and \( \overline{JK} \) intersect at \( M \). Find the value of \( x \) and the measure of \( \angle JMH \).

\[
\begin{align*}
16x - 20 &= 13x + 7 \\
3x - 20 &= 7 \\
\sqrt{3}x &= 27 \\
\sqrt{3} &= 3 \\
x &= 9
\end{align*}
\]

\[m\angle JMH = 180 - 124 \]
\[m\angle JMH = 56^\circ\]
Example 5
Suppose \( m \angle GMJ = 9x - 4 \) and \( m \angle MH = 4x - 11 \). Find the value of \( x \) and \( m \angle KMH \).

\[
\begin{align*}
9x - 4 + 4x - 11 &= 180 \\
13x - 15 &= 180 \\
13x &= 195 \\
x &= 15
\end{align*}
\]

Two angles whose measures have a sum of 180 are called **supplementary** angles. If the sum of their measures is 90, they are called **complementary** angles.

Since we have learned that the sum of the measures of a linear pair is 180, we can now say that any two angles that form a linear pair must be supplementary angles.
Example 6
a) Name a pair of complementary angles.
\( \angle CAB \ & \ & \angle RST \)
b) Name a pair of supplementary angles.
\( \angle CAD \ & \ & \angle RST \)
c) Name a pair of adjacent angles.
\( \angle CAB \ & \ & \angle DAC \)

Example 7
a) Given that \( \angle 1 \) is a complement of \( \angle 2 \) and \( m\angle 1 = 62^\circ \), find \( m\angle 2 \).
\[ m\angle 2 = 28^\circ \]

b) Given that \( \angle 3 \) is a supplement of \( \angle 4 \) and \( m\angle 4 = 114^\circ \), find \( m\angle 3 \).
\[ m\angle 3 = 66^\circ \]
Example 8
\( \angle LMN \) and \( \angle PQR \) are complementary angles. Find the measures of the angles if \( m \angle LMN = (4x - 2) \) and \( m \angle PQR = (9x + 1) \)°.

\[
(4x - 2) + (9x + 1) = 90 \\
13x - 1 = 90 \\
13x = 91 \\
x = 7
\]

\( m \angle LMN = 4(7) - 2 = 26 \)°

\( m \angle PQR = 9(7) + 1 = 64 \)°

Example 9
Two angles form a linear pair. The measure of one angle is 5 times the measure of the other. Find the measure of each angle.

\[
180 = 5x + x \\
180 = 6x \\
30 = x
\]

\( 30° \) & \( 150° \)
Example 10
Two angles are complementary. One angle is six less than twice the other angle. Find the measure of each angle.

\[
\text{angle 1} \quad + \quad \text{angle 2} = 90
\]
\[
2x - 6 \quad + \quad x = 90
\]
\[
3x - 6 = 90
\]
\[
3x = 96
\]
\[
x = 32
\]

[32° & 58°]