1.5 Equations

solutions = roots = x-intercepts = zeros

Linear Equation: first degree equation

Example 1: Solve.

$$
\begin{aligned}
7x - 4 &= 3x + 8 \\
-3x \quad & \quad -3x \\
4x - 4 &= 8 \\
+4 \quad & \quad +4 \\
\frac{4x}{4} &= \frac{12}{4} \\
\boxed{x = 3}
\end{aligned}
$$

Literal Equation: equation with several variables

Example 2:

Solve for M.

$$r^2 \cdot F = G \cdot \frac{mM}{r^2} \cdot r^2$$

$$\frac{Fr^2}{Gm} = \frac{GmM}{Gm}$$

$$\boxed{\frac{Fr^2}{Gm} = M}$$

Example 3:

Solve for w.

$$
\begin{aligned}
A &= 2lw + 2wh + 2lh \\
-2lh \quad & \quad -2lh \\
A - 2lh &= 2lw + 2wh \\
\frac{A-2lh}{2l+2h} &= \frac{w(2l+2h)}{2l+2h} \\
\boxed{\frac{A-2lh}{2l+2h} = w}
\end{aligned}
$$

Quadratic Equation: 2nd degree equations

Example 4:
Solve by factoring.

$x^2 + 5x = 24$
$-24 \quad -24$
$x^2+5x-24=0$
$(x+8)(x-3)=0$
$x+8=0 \qquad x-3=0$
$\boxed{x=-8} \qquad \boxed{x=3}$

Example 5:
Solve.

$x^2 = 25$
$-25 \quad -25$
$x^2-25=0$
$(x-5)(x+5)=0$
$x-5=0 \qquad x+5=0$
$\boxed{x=5} \qquad \boxed{x=-5}$

OR
$\sqrt{x^2}=\sqrt{25}$
$\boxed{x=\pm 5}$

Example 6:
ax^2+bx+c
Solve by completing the square.

$x^2 - 8x + 13 = 0$
$-13 \quad -13$
$x^2-8x = -13$
Find ½ of b. $\frac{1}{2}(-8)=-4$
Square answer. $(-4)^2=16$
Add answer to each side.
perf. sq. trinomial
$x^2-8x+16=-13+16$
$\sqrt{(x-4)^2}=\sqrt{3}$
$x-4=\pm\sqrt{3}$
$+4 \quad +4$
$\boxed{x=4\pm\sqrt{3}}$

Example 7:
Solve by completing the square.

$3x^2 - 12x + 6 = 0$
$-6 \quad -6$
$3x^2-12x = -6$
$3(x^2-4x \quad)=-6$
$\frac{1}{2}(-4)=-2$
$(-2)^2=4$
$3(x^2-4x+4)=-6+12$
$\frac{3(x-2)^2}{3}=\frac{6}{3}$
$\sqrt{(x-2)^2}=\sqrt{2}$
$x-2=\pm\sqrt{2}$
$+2 \quad +2$
$\boxed{x=2\pm\sqrt{2}}$

Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Example 8:

Solve using the quadratic formula.

$2|48$, $2|24$, $2|12$, $2|6$, 3

$a=3 \quad b=-6 \quad c=-1$

$3x^2 - 6x - 1 = 0$

$x = \dfrac{6 \pm \sqrt{(-6)^2 - 4(3)(-1)}}{2(3)}$

$x = \dfrac{6 \pm \sqrt{36+12}}{6}$

$x = \dfrac{6 \pm \sqrt{48}}{6}$

$x = \dfrac{6 \pm 4\sqrt{3}}{6}$

$x = \dfrac{2(3 \pm 2\sqrt{3})}{6 \div 2} \div 2$

$\boxed{x = \dfrac{3 \pm 2\sqrt{3}}{3}}$

$x = \dfrac{6}{6} \pm \dfrac{4\sqrt{3}}{6}$

$\boxed{x = 1 \pm \dfrac{2\sqrt{3}}{3}}$

Example 9:

Solve using the quadratic formula.

$a=4 \quad b=12 \quad c=9$

$4x^2 + 12x + 9 = 0$

$x = \dfrac{-12 \pm \sqrt{(12)^2 - 4(4)(9)}}{2(4)}$

$x = \dfrac{-12 \pm \sqrt{144-144}}{8}$

$x = \dfrac{-12 \cancel{\pm \sqrt{0}}}{8}$

$\boxed{x = -\dfrac{3}{2}}$

Example 10:

Solve using the quadratic formula.

$x^2 + 2x = -2$

$\quad +2 \quad +2$

$x^2 + 2x + 2 = 0$

$x = \dfrac{-2 \pm \sqrt{(2)^2 - 4(1)(2)}}{2(1)}$

$x = \dfrac{-2 \pm \sqrt{4-8}}{2}$

$x = \dfrac{-2 \pm \sqrt{-4}}{2}$

$x = \dfrac{-2 \pm 2i}{2} = \dfrac{\cancel{2}(-1 \pm i)}{\cancel{2}}$

$\boxed{x = -1 \pm i}$

Discriminant: a real number that tells how many and what type of solutions a quadratic equation has

1. If $D > 0$, then 2 real solutions.
2. If $D = 0$, then 1 real solution.
3. If $D < 0$, then no real solutions (2 complex).

Example 11:

Determine how many real solutions each equation has.

a) $x^2 + 4x - 1 = 0$

$b^2-4ac=(4)^2-4(1)(-1)$
$=16+4$
$=20$
2 real solutions

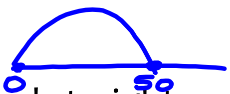
b) $4x^2 - 12x = -9$

$+9 \quad +9$
$4x^2-12x+9=0$
$b^2-4ac=(-12)^2-4(4)(9)$
$=144-144$
$=0$
1 real solution

c) $\frac{1}{3}x^2 - 2x + 4 = 0$

$b^2-4ac=(-2)^2-4(\frac{1}{3})(4)$
$=4-\frac{16}{3}$
$=\frac{12}{3}-\frac{16}{3}$
$=-\frac{4}{3}$
no real solutions

Example 12:



An object is thrown or fired straight upward at an initial speed of v_0 ft/s and will reach a height of h feet after t seconds, where h and t are related by the formula $h = -16t^2 + v_0t$. Suppose that a bullet is shot straight upward with an initial speed of 800 ft/s.

v_0

$h=-16t^2+800t$

a) When does the bullet fall back to ground level?

$h=0$

$0=-16t^2+800t$
$0=-16t(t-50)$

beg. of problem

$\frac{-16t}{-16}=\frac{0}{-16}$ $\quad$ $t-50=0$

$t \neq 0$ $\quad$ $\boxed{t=50 \text{ sec}}$

b) When does it reach a height of 6400 ft?

h

$6400=-16t^2+800t$

$16t^2-800t+6400=0$
$16(t^2-50t+400)=0$
$16(t-10)(t-40)=0$
$16\neq 0 \quad t-10=0 \quad t-40=0$
$\boxed{t=10 \text{ sec}} \quad \boxed{t=40 \text{ sec}}$

Example 12 (continued):

1 mile = 5280 ft
2 miles = 10560 ft

c) When does it reach a height of 2 miles?

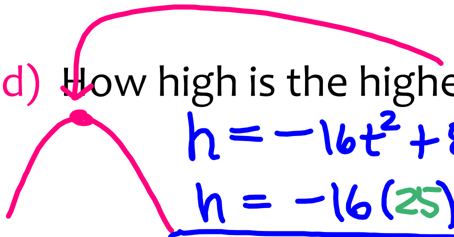
$10560 = -16t^2 + 800t$ ↑ NEVER

$16t^2 - 800t + 10560 = 0$
$16(t^2 - 50t + 660) = 0$

$b^2 - 4ac = (-50)^2 - 4(1)(660)$
$= 2500 - 2640$

neg ans ⟶ no real solutions

d) How high is the highest point the bullet reaches?



$h = -16t^2 + 800t$ $t = \frac{-b}{2a} = \frac{-800}{-32} = 25$

$h = -16(25)^2 + 800(25)$

$\boxed{h = 10{,}000 \text{ ft}}$

(Rational) Fractional Equations: Check for extraneous solutions.

Example 13: Solve.

Find LCD & multiply on both sides.

LCD : $x(x+2)$

denom $\neq 0$

$$\frac{3}{x} + \frac{5}{x+2} = 2$$

$$\cancel{x}(x+2) \cdot \frac{3}{\cancel{x}} + x\cancel{(x+2)} \cdot \frac{5}{\cancel{x+2}} = x(x+2) \cdot 2$$

$3(x+2) + 5x = 2x(x+2)$
$3x + 6 + 5x = 2x^2 + 4x$
$-3x \; -6 \quad -5x \qquad -3x - 5x - 6$

$0 = 2x^2 - 4x - 6$
$0 = 2(x^2 - 2x - 3)$
$0 = 2(x-3)(x+1)$

$x - 3 = 0$ $x + 1 = 0$
$\boxed{x = 3}$ $\boxed{x = -1}$

Radical Equations: Check for extraneous solutions.

Isolate radical.

Example 14: Solve.

$2x = 1 - \sqrt{2 - x}$

$-1 \quad -1$

$2x - 1 = -\sqrt{2-x}$

-1 $\quad$ -1

$(1-2x)^2 = (\sqrt{2-x})^2$

$(1-2x)(1-2x)$

$1 - 4x + 4x^2 = 2 - x$

$-2 \quad +x \qquad -2 + x$

$4x^2 - 3x - 1 = 0$

$(x-1)(4x+1) = 0$

$x - 1 = 0 \qquad 4x + 1 = 0$

$\cancel{x = 1} \qquad \boxed{x = -\frac{1}{4}}$

$\frac{-1}{1} \cancel{\frac{-4}{4}} \quad \frac{1}{4}$

Check: $x = 1$

$2(1) \stackrel{?}{=} 1 - \sqrt{2-1}$

$2 = 1 - 1$

$2 \neq 0$

Check: $x = -\frac{1}{4}$

$2(-\frac{1}{4}) \stackrel{?}{=} 1 - \sqrt{2 - (-\frac{1}{4})}$

$-\frac{1}{2} = 1 - \sqrt{\frac{9}{4}}$

$-\frac{1}{2} = 1 - \frac{3}{2}$

$-\frac{1}{2} = -\frac{1}{2}$ ✓

Fourth - Degree Equation of Quadratic Type

$x^2 - 8x + 8 = 0$

Example 15: Solve.

$a = 1 \quad b = -8 \quad c = 8$

$x^4 - 8x^2 + 8 = 0$

$x^2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x^2 = \frac{8 \pm \sqrt{(-8)^2 - 4(1)(8)}}{2(1)}$

$x^2 = \frac{8 \pm \sqrt{32}}{2}$

$x^2 = \frac{8 \pm 4\sqrt{2}}{2}$

$x^2 = \frac{\cancel{2}(4 \pm 2\sqrt{2})}{\cancel{2}}$

$\sqrt{x^2} = \sqrt{4 \pm 2\sqrt{2}}$

$\boxed{x = \pm\sqrt{4 \pm 2\sqrt{2}}}$

Equations with Fractional Powers: Check for extraneous solutions.

Example 16: Solve.

$x^{\frac{1}{3}} + x^{\frac{1}{6}} - 2 = 0$ sum 1 prod −2

$\frac{2}{1}\ \frac{-1}{1}$

$(x^{\frac{1}{6}} + 2)(x^{\frac{1}{6}} - 1) = 0$

$(\sqrt[6]{x} + 2)(\sqrt[6]{x} - 1) = 0$

$\sqrt[6]{x} + 2 = 0$ $\sqrt[6]{x} - 1 = 0$

$(\sqrt[6]{x})^6 = (-2)^6$ $(\sqrt[6]{x})^6 = (1)^6$

$x \neq 64$ $\boxed{x = 1}$

Check: $x = 64$

$\sqrt[3]{64} + \sqrt[6]{64} - 2 \stackrel{?}{=} 0$

$4 + 2 - 2 \neq 0$

$4 \neq 0$

Check: $x = 1$

$\sqrt[3]{1} + \sqrt[6]{1} - 2 \stackrel{?}{=} 0$

$1 + 1 - 2 = 0$ ✓

Absolute Value Equations: Remember what absolute value means.

Example 17: Solve.

dist. is 3

$|2x - 5| = 3$

$2x - 5 = -3$ or $2x - 5 = 3$

$2x = 2$ $2x = 8$

$\boxed{x = 1}$ $\boxed{x = 4}$

Example 18: Solve.

dist. is −1

$|3x + 4| = -1$

no solution

distance isn't negative