

## 2.4 OPERATIONS WITH FUNCTIONS

For all functions  $f$  and  $g$ :

**Sum**  $(f + g)(x) = f(x) + g(x)$

**Difference**  $(f - g)(x) = f(x) - g(x)$

**Product**  $(f \cdot g)(x) = f(x) \cdot g(x)$

**Quotient**  $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ , where  $g(x) \neq 0$

Let  $f(x) = 4x^2 + 6x - 9$  and  $g(x) = 6x^2 - x + 2$ .

1. Find  $f + g$ .

$$f + g = (4x^2 + 6x - 9) + (6x^2 - x + 2)$$

$$f + g = 10x^2 + 5x - 7$$

2. Find  $f - g$ .

$$f - g = (4x^2 + 6x - 9) - (6x^2 - x + 2)$$
$$f - g = 4x^2 + 6x - 9 - 6x^2 + x - 2$$

$$f - g = -2x^2 + 7x - 11$$

$$\text{Let } f(x) = 9x^2 \text{ and } g(x) = 4x + 3.$$

3. Find  $f \cdot g$ .

$$f \cdot g = 9x^2 (4x + 3)$$

$$f \cdot g = 36x^3 + 27x^2$$

4. Find  $\frac{f}{g}$ , and state ~~any~~ any domain restrictions.

$$\frac{f}{g} = \frac{9x^2}{4x+3}$$

denominator  $\neq 0$

$$4x + 3 \neq 0$$

$$\frac{4x}{4} \neq \frac{-3}{4}$$

$$x \neq -\frac{3}{4}$$

domain  
restriction

## COMPOSITION OF FUNCTIONS

Let  $f$  and  $g$  be functions of  $x$ .

The composition of  $f$  with  $g$ ,  
denoted  $f \circ g$ ,  
is defined by  $f(g(x))$ .

*outside function* (pointing to  $f$ )  
*inside function* (pointing to  $g$ )

Let  $f(x) = x^2 - 1$  and  $g(x) = 3x$ .

5. Find  $f \circ g$ .

$$f(g(x)) = (3x)^2 - 1$$

$$f \circ g = 9x^2 - 1$$

6. Find  $g \circ f$ .

$$g(f(x)) = 3(x^2 - 1)$$

$$g \circ f = 3x^2 - 3$$

Let  $f(x) = -2x^2 + 3$  and  $g(x) = -2x$ .

7. Find  $f \circ g$ .

$$\begin{aligned} f(g(x)) &= -2(-2x)^2 + 3 \\ &= -2(4x^2) + 3 \end{aligned}$$

$$\boxed{f \circ g = -8x^2 + 3}$$

8. Find  $g \circ f$ .

$$g(f(x)) = -2(-2x^2 + 3)$$

$$\boxed{g \circ f = 4x^2 - 6}$$

Let  $f(x) = 2x$ ,  $g(x) = x^2 + 2$ , and  $h(x) = -4x + 3$ .

9. Find  $(f \circ h)(3) = -18$

$$\begin{aligned} f(h(3)) &= f(-9) \\ &= 2(-9) \\ &= -18 \end{aligned}$$

$$\begin{aligned} h(3) &= -4(3) + 3 \\ h(3) &= -12 + 3 \\ h(3) &= -9 \end{aligned}$$

10. Find  $(h \circ g)(-2) = -21$

$$\begin{aligned} h(g(-2)) &= h(6) \\ &= -4(6) + 3 \\ &= -24 + 3 \\ &= -21 \end{aligned}$$

$$\begin{aligned} g(-2) &= (-2)^2 + 2 \\ g(-2) &= 4 + 2 \\ g(-2) &= 6 \end{aligned}$$

11. Find  $(f \circ h) \circ g$ .

START

$$\begin{aligned} f(h(x)) &= 2(-4x+3) \\ &= -8x+6 \end{aligned}$$

$$\begin{aligned} \underbrace{(f \circ h)}_{\text{outside}} \circ \underbrace{g}_{\text{inside}} &= -8x^2 - 10 \\ -8(x^2 + 2) + 6 & \\ -8x^2 - 16 + 6 & \\ -8x^2 - 10 & \end{aligned}$$