1.5 Equations

solutions = roots = x-intercepts = zeros

Linear Equation: first degree equation

Example 1: Solve.

$$7x - 4 = 3x + 8$$
$$-3x \quad -3x$$
$$4x - 4 = 8$$
$$+4 \quad +4$$
$$\frac{4x}{4} = \frac{12}{4}$$
$$\boxed{x = 3}$$

Literal Equation: equation with several variables

Example 2:

Solve for M.

$$r^2 \cdot F = G \cdot \frac{mM}{r^2} \cdot r^2$$
$$\frac{Fr^2}{Gm} = \frac{GmM}{Gm}$$
$$\boxed{\frac{Fr^2}{Gm} = M}$$

Example 3:

Solve for w.

$$A = 2lw + 2wh + 2lh$$
$$-2lh \qquad -2lh$$
$$A - 2lh = 2lw + 2wh$$
$$\frac{A - 2lh}{2l + 2h} = \frac{w(2l + 2h)}{2l + 2h}$$
$$\boxed{\frac{A - 2lh}{2l + 2h} = w}$$

Quadratic Equation: 2nd degree equations

Example 4:
Solve by factoring.

$x^2 + 5x = 24$

$-24 \quad -24$

$x^2+5x-24=0$

$(x+8)(x-3)=0$

$x+8=0 \qquad x-3=0$

$\boxed{x=-8} \qquad \boxed{x=3}$

Example 5:
Solve.

$x^2 = 25$

$-25 \quad -25$

$x^2-25=0$

$(x-5)(x+5)=0$

$x-5=0 \qquad x+5=0$

$\boxed{x=5} \qquad \boxed{x=-5}$

OR

$\sqrt{x^2}=\sqrt{25}$

$\boxed{x=\pm 5}$

Example 6:
ax^2+bx+c

Solve by completing the square.

$x^2 - 8x + 13 = 0$

$-13 \quad -13$

$x^2-8x \qquad = -13$

Find ½ of b. $\frac{1}{2}(-8)=-4$

Square answer. $(-4)^2=16$

Add answer to each side.

perf. sq. trinomial

$x^2-8x+16=-13+16$

$\sqrt{(x-4)^2}=\sqrt{3}$

$x-4=\pm\sqrt{3}$

$+4 \quad +4$

$\boxed{x=4\pm\sqrt{3}}$

Example 7:
Solve by completing the square.

$3x^2 - 12x + 6 = 0$

$-6 \quad -6$

$3x^2-12x \qquad = -6$

$3(x^2-4x \qquad) = -6$

$\frac{1}{2}(-4)=-2$

$(-2)^2=4$

$3(x^2-4x+4)=-6+12$

$\frac{3(x-2)^2}{3}=\frac{6}{3}$

$\sqrt{(x-2)^2}=\sqrt{2}$

$x-2=\pm\sqrt{2}$

$+2 \quad +2$

$\boxed{x=2\pm\sqrt{2}}$

Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Example 8:

Solve using the quadratic formula.

$a=3 \quad b=-6 \quad c=-1$

$3x^2 - 6x - 1 = 0$

$x = \dfrac{6 \pm \sqrt{(-6)^2 - 4(3)(-1)}}{2(3)}$

$x = \dfrac{6 \pm \sqrt{36+12}}{6}$

$x = \dfrac{6 \pm \sqrt{48}}{6}$

2 | 48, 2 | 24, 2 | 12, 2 | 6, 3

$x = \dfrac{6 \pm 4\sqrt{3}}{6}$

$x = \dfrac{2(3 \pm 2\sqrt{3}) \div 2}{6 \div 2}$

$\boxed{x = \dfrac{3 \pm 2\sqrt{3}}{3}}$

$x = \dfrac{6}{6} \pm \dfrac{4\sqrt{3}}{6}$

$\boxed{x = 1 \pm \dfrac{2\sqrt{3}}{3}}$

Example 9:

Solve using the quadratic formula.

$a=4 \quad b=12 \quad c=9$

$4x^2 + 12x + 9 = 0$

$x = \dfrac{-12 \pm \sqrt{(12)^2 - 4(4)(9)}}{2(4)}$

$x = \dfrac{-12 \pm \sqrt{144-144}}{8}$

$x = \dfrac{-12 \cancel{\pm 0}}{8}$

$\boxed{x = -\dfrac{3}{2}}$

Example 10:

Solve using the quadratic formula.

$x^2 + 2x = -2$

$\quad\; +2 \quad +2$

$x^2 + 2x + 2 = 0$

$x = \dfrac{-2 \pm \sqrt{(2)^2 - 4(1)(2)}}{2(1)}$

$x = \dfrac{-2 \pm \sqrt{4-8}}{2}$

$x = \dfrac{-2 \pm \sqrt{-4}}{2}$

$x = \dfrac{-2 \pm 2i}{2} = \dfrac{\cancel{2}(-1 \pm i)}{\cancel{2}}$

$\boxed{x = -1 \pm i}$

Discriminant: a real number that tells how many and what type of solutions a quadratic equation has

1. If $D > 0$, then 2 real solutions.
2. If $D = 0$, then 1 real solution.
3. If $D < 0$, then no real solutions (2 complex).

Example 11:

Determine how many real solutions each equation has.

a) $x^2 + 4x - 1 = 0$

$b^2-4ac=(4)^2-4(1)(-1)$
$=16+4$
$=20$
2 real solutions

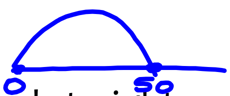
b) $4x^2 - 12x = -9$

$+9 \quad +9$
$4x^2-12x+9=0$
$b^2-4ac=(-12)^2-4(4)(9)$
$=144-144$
$=0$
1 real solution

c) $\frac{1}{3}x^2 - 2x + 4 = 0$

$b^2-4ac=(-2)^2-4(\frac{1}{3})(4)$
$=4-\frac{16}{3}$
$=\frac{12}{3}-\frac{16}{3}$
$=-\frac{4}{3}$
no real solutions

Example 12:



An object is thrown or fired straight upward at an initial speed of v_0 ft/s and will reach a height of h feet after t seconds, where h and t are related by the formula $h = -16t^2 + v_0t$. Suppose that a bullet is shot straight upward with an initial speed of 800 ft/s.

v_0

$h=-16t^2+800t$

a) When does the bullet fall back to ground level?

$h=0$

$0=-16t^2+800t$
$0=-16t(t-50)$

beg. of problem

$\frac{-16t}{-16}=\frac{0}{-16}$ $\quad$ $t-50=0$
$\cancel{t=0}$ $\quad$ $\boxed{t=50 \text{ sec}}$

b) When does it reach a height of 6400 ft?

h

$6400=-16t^2+800t$
$16t^2-800t+6400=0$
$16(t^2-50t+400)=0$
$16(t-10)(t-40)=0$
$16\neq 0$ $\quad$ $t-10=0$ $\quad$ $t-40=0$
$\boxed{t=10 \text{ sec}}$ $\quad$ $\boxed{t=40 \text{ sec}}$

Example 12 (continued):

1 mile = 5280 ft
2 miles = 10560 ft

c) When does it reach a height of 2 miles?
h

$10560 = -16t^2 + 800t$

$16t^2 - 800t + 10560 = 0$

$16(t^2 - 50t + 660) = 0$

d) How high is the highest point the bullet reaches?