2.5 INVERSES OF FUNCTIONS

The inverse of a relation consisting of the ordered pairs (x,y) is the set of all ordered pairs (y,x).

The domain of the inverse is the range of the original relation.

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EXAMPLES: Find the inverse of each relation. State whether the relation is a function. State whether the inverse is a function.

1. $\{(1, 2), (2, 4), (3, 6), (4, 8)\}$ — R: function

$\{(2,1), (4,2), (6,3), (8,4)\}$ — function

2. $\{(1, 5), (1, 6), (3, 6), (4, 9)\}$ — R: not a function

$\{(5,1), (6,1), (6,3), (9,4)\}$ — not a function

To find the inverse of a function, simply interchange x and y, and then solve for y.

3. Find an equation for the inverse of $y = 3x - 2$.

$x = 3y - 2$

$+2 \qquad +2$

$\frac{x+2}{3} = \frac{3y}{3}$ or $\frac{x}{3} + \frac{2}{3} = \frac{3y}{3}$

$\frac{x+2}{3} = y$ or $\frac{1}{3}x + \frac{2}{3} = y$

4. Find an equation for the inverse of $y = 4x + 5$.

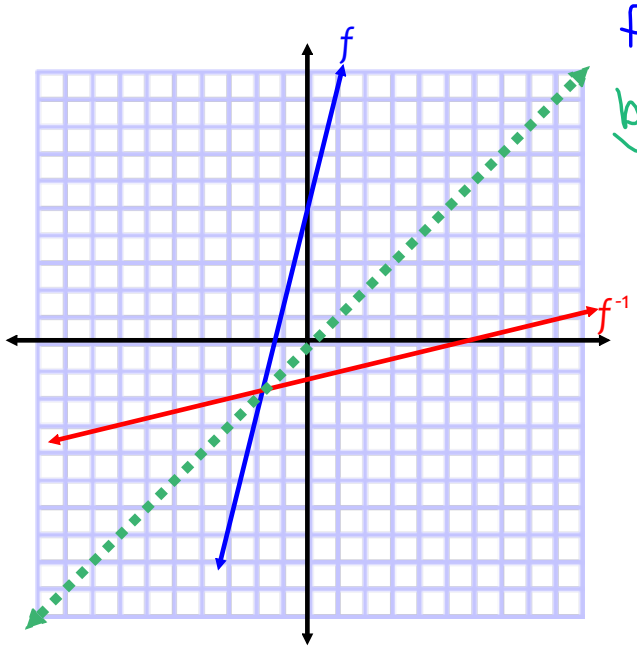
$x = 4y + 5$

$-5 \qquad -5$

$\frac{x-5}{4} = \frac{4y}{4}$

$\frac{x-5}{4} = y$

If a function f and its inverse are **both functions**, the inverse of f is denoted by f^{-1}.



f inverse

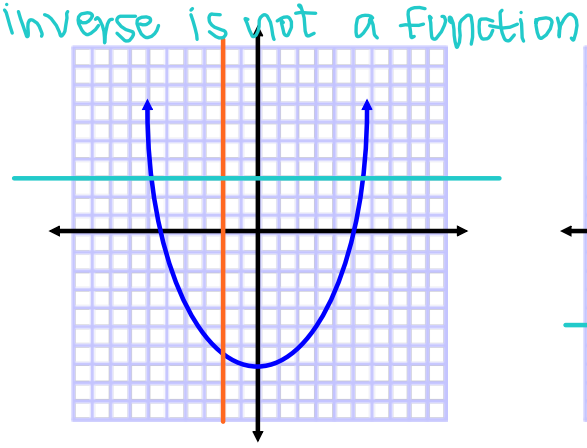
b=0 m=1

y=x

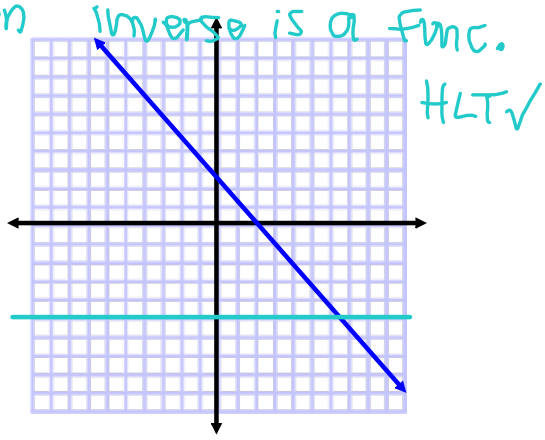
HORIZONTAL LINE TEST

The inverse of a function is a function if and only if every horizontal line intersects the graph of the given function at no more than one point.

function
inverse is not a function



function VLT✓
inverse is a func.
HLT✓



If a function has an inverse that is also a function, then the function is **one-to-one**.

VLT✓ HLT✓

Every one-to-one function passes **both** the vertical line test and the horizontal line test.

COMPOSITION AND INVERSES

If f and g are functions and

$(f \circ g)(x) = (g \circ f)(x) = x$,

then f and g are inverses of one another.

5. Show that $f(x) = 4x - 3$ and $g(x) = \frac{1}{4}x + \frac{3}{4}$ are inverses of each other.

$f \circ g$

$4\left(\frac{1}{4}x + \frac{3}{4}\right) - 3$

$x + 3 - 3$

x ✓

$g \circ f$

$\frac{1}{4}(4x - 3) + \frac{3}{4}$

$x - \frac{3}{4} + \frac{3}{4}$

x ✓

6. Show that $f(x) = -5x + 7$ and $g(x) = -\frac{1}{5}x + \frac{7}{5}$ are inverses of each other.

$f \circ g$

$-5\left(-\frac{1}{5}x + \frac{7}{5}\right) + 7$

$x - 7 + 7$

x ✓

$g \circ f$

$-\frac{1}{5}(-5x + 7) + \frac{7}{5}$

$x - \frac{7}{5} + \frac{7}{5}$

x ✓