1.6 Modeling with Equations

Example 1

Find four consecutive odd integers whose sum is 272.

Let x = 1st cons. odd int.

$x+2$ = 2nd

$x+4$ = 3rd

$x+6$ = 4th

65
67
69
71

$$x+(x+2)+(x+4)+(x+6)=272$$

$$4x+12=272$$

$$-12 \quad -12$$

$$\frac{4x}{4}=\frac{260}{4}$$

$$x=65$$

Example 2

The sum of the squares of two consecutive even integers is 1252. Find the integers.

Let x = 1st cons. even int.

$x+2$ = 2nd

24
26

−26
−24

$$x^2+(x+2)^2=1252$$

$$x^2+x^2+4x+4=1252$$

$$2x^2+4x+4=1252$$

$$-1252 \quad -1252$$

$$2x^2+4x-1248=0$$

S2 p-624

$$2(x^2+2x-624)=0$$

$$x=\frac{-2\pm\sqrt{(2)^2-4(1)(-624)}}{2(1)}$$

$$x=\frac{-2\pm\sqrt{2500}}{2}$$

$$x=\frac{-2\pm 50}{2}$$

$$\frac{-2+50}{2}=\frac{48}{2}=24$$

$$\frac{-2-50}{2}=\frac{-52}{2}=-26$$

$I = prt$ Simple Interest

Example 3

Mary inherits $100,000 and invests it in two certificates of deposit. One certificate pays 6% and the other pays 4.5% simple interest annually. If Mary's total interest is $5025 per year, how much money is invested at each rate?

first cert.
$p = x$
$r = .06$
$t = 1$

Second cert.
$p = 100{,}000 - x$
$r = .045$
$t = 1$

$(x)(.06)(1) + (100{,}000 - x)(.045)(1) = 5025$

$.06x + 4500 - .045x = 5025$

$.015x + 4500 = 5025$

$-4500 \quad -4500$

$\frac{.015x}{.015} = \frac{525}{.015}$

$x = 35{,}000$

$35,000 @ 6% rate
$65,000 @ 4.5% rate

Example 4

If Ben invests $4000 at 4% interest per year, how much additional money must he invest at 5.5% annual interest to ensure that the interest he receives each year is 4.5% of the total amount invested?

4% Rate 5.5% Rate 4.5% of Total Invested

$(4000)(.04)(1) + (x)(.055)(1) = .045(4000 + x)$

$160 + .055x = 180 + .045x$

$-.045x \quad -.045x$

$160 + .01x = 180$

$-160 \quad -160$

$\frac{.01x}{.01} = \frac{20}{.01}$

$x = 2000$

$2000 at 5.5%

Example 5

Jack invests $1000 at a certain annual interest rate, and he invests another $2000 at an annual rate that is one-half percent higher. If he receives a total of $190 interest in one year, at what rate is the $1000 invested?

½% higher .5%

investment 1 + investment 2 = 190

$(1000)(x)(1) + (2000)(x+.005)(1) = 190$

$1000x + 2000x + 10 = 190$

$3000x + 10 = 190$

$\frac{3000x}{3000} = \frac{180}{3000}$

$x = .06$

6% rate

Example 6

A woman earns 15% more than her husband. Together they make $69,875 per year. What is the husband's annual salary?

husband earns = x

wife earns = $x + .15x = 1.15x$

$x + 1.15x = 69,875$

$\frac{2.15x}{2.15} = \frac{69,875}{2.15}$

$x = 32,500$

$32,500 = husband's salary

Example 7

Helen earns $7.50 per hour at her job, but if she works more than 35 hours in a week she is paid 1.5 times her regular salary for the overtime hours worked. One week her gross pay was $352.50. How many overtime hours did she work that week?

x = # of extra hours

$(7.50)(35) + (1.5 \times 7.50)(x) = 352.50$

$262.50 + 11.25x = 352.50$

$-262.50 \quad -262.50$

$\frac{11.25x}{11.25} = \frac{90}{11.25}$

$x = 8$

8 overtime hours

Example 8

A father is four times as old as his daughter. In 6 years, he will be three times as old as she is. How old is the daughter now?

NOW

x = daughter

$4x$ = father

SIX YEARS LATER

daughter = $x+6$

father = $3(x+6)$

father in 6 years

$4x + 6 = 3(x+6)$

$4x + 6 = 3x + 18$

$x + 6 = 18$

$x = 12$

daughter is 12 years old

Example 9

Mary has \$3.00 in nickels, dimes, and quarters. If she has twice as many dimes as quarters and five more nickels than dimes, how many coins of each type does she have?

2x+5 (nickels), 2x (dimes), x (quarters)

$.05(2x+5) + .10(2x) + .25(x) = 3.00$

$.1x + .25 + .2x + .25x = 3.00$

$.55x + .25 = 3.00$

$-.25 \quad -.25$

$\frac{.55x}{.55} = \frac{2.75}{.55}$

$x = 5$

5 quarters
10 dimes
15 nickels

Example 10

A pasture is twice as long as it is wide. Its area is 115,200 ft^2. How wide is the pasture?

2x (long), x (wide)

$A = lw$

$115{,}200 = (2x)(x)$

$\frac{115{,}200}{2} = \frac{2x^2}{2}$

$\sqrt{57{,}600} = \sqrt{x^2}$

$240 = x$

width is 240 ft

Example 11

A square plot of land has a building 60 feet long and 40 feet wide at one corner. The rest of the land outside the building forms a parking lot. If the parking lot has area 12,000 ft^2, what are the dimensions of the entire plot of land?

square − building = parking

$x^2 - (60)(40) = 12{,}000$

$x^2 - 2400 = 12{,}000$

$+2400 \quad +2400$

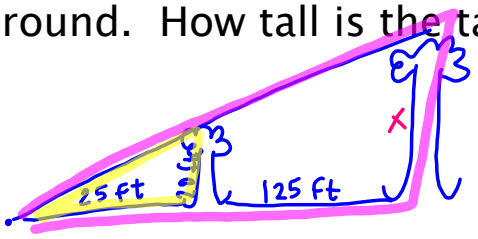
$\sqrt{x^2} = \sqrt{14{,}400}$

$x = 120$

120 ft by 120 ft

Example 12

A woodcutter determines the height of a tall tree by first measuring a smaller one 125 feet away, then moving so that his eyes are in the line of sight along the tops of the trees, and measuring how far he is standing from the small tree. Suppose the small tree is 20 feet tall, the man is 25 feet from the small tree, and his eye level is 5 feet above the ground. How tall is the taller tree?



$\frac{25}{20} = \frac{150}{x}$

$\frac{25x}{25} = \frac{3000}{25}$

$x = 120$

120 ft

$d = rt$

Example 13

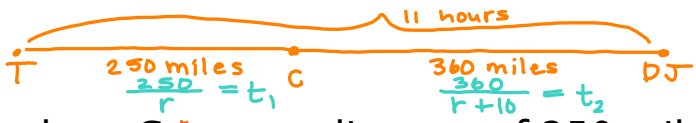
Two cyclists, 90 miles apart, start riding toward each other at the same time. One cycles twice as fast as the other. If they meet 2 hours later, at what average speed is each cyclist traveling?

$$d_1 + d_2 = 90$$
$$2r + 4r = 90$$
$$6r = 90$$
$$r = 15$$

cyclist = 15 mph
cyclist = 30 mph

$d = rt$ $\quad \frac{d}{r} = t$

Example 14



Kate drove from Tortula to Cactus, a distance of 250 miles. She increased her speed by 10 mph for the 360-mile trip from Cactus to Dry Junction. If the total trip took 11 hours, what was her speed for Tortula to Cactus?

$t_1 + t_2 = 11$ $\quad$ LCD: $r(r+10)$

$$\frac{250}{r} + \frac{360}{r+10} = 11$$
$$r(r+10)\cdot\frac{250}{r} + r(r+10)\cdot\frac{360}{r+10} = r(r+10)\cdot 11$$
$$250(r+10) + 360r = 11r(r+10)$$
$$250r + 2500 + 360r = 11r^2 + 110r$$
$$2500 + 610r = 11r^2 + 110r$$
$$-2500 \quad -610r \qquad -610r \quad -2500$$
$$0 = 11r^2 - 500r - 2500$$
$$r = \frac{500 \pm \sqrt{(-500)^2 - 4(11)(-2500)}}{2(11)}$$
$$r = \frac{500 \pm \sqrt{360{,}000}}{22}$$
$$r = \frac{500 \pm 600}{22}$$
$$\frac{500+600}{22} = \frac{1100}{22} = 50$$
$$\frac{500-600}{22} = \frac{-100}{22}$$

50 mph from Tortula to Cactus