1.6 Modeling with Equations

Example 1

Find four consecutive odd integers whose sum is 272.

Let x = 1st cons. odd int.

$x + 2$ = 2nd
$x + 4$ = 3rd
$x + 6$ = 4th

65
67
69
71

$$x + (x+2) + (x+4) + (x+6) = 272$$
$$4x + 12 = 272$$
$$ -12 \qquad -12$$
$$\frac{4x}{4} = \frac{260}{4}$$
$$x = 65$$

Example 2

The sum of the squares of two consecutive even integers is 1252. Find the integers.

Let x = 1st cons. even int.

$x + 2$ = 2nd

24
26

-26
-24

$$x^2 + (x+2)^2 = 1252$$
$$x^2 + x^2 + 4x + 4 = 1252$$
$$2x^2 + 4x + 4 = 1252$$
$$ -1252 \qquad -1252$$
$$2x^2 + 4x - 1248 = 0 \qquad \underline{52 \quad p-624}$$
$$2(x^2 + 2x - 624) = 0$$
$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-624)}}{2(1)}$$
$$x = \frac{-2 \pm \sqrt{2500}}{2}$$
$$x = \frac{-2 \pm 50}{2} \qquad \frac{-2+50}{2} = \frac{48}{2} = 24$$
$$\searrow \frac{-2-50}{2} = \frac{-52}{2} = -26$$

$I = prt$ Simple Interest

Example 3

Mary inherits $100,000 and invests it in two certificates of deposit. One certificate pays 6% and the other pays 4.5% simple interest annually. If Mary's total interest is $5025 per year, how much money is invested at each rate?

first cert.
p= x
r= .06
t = 1

Second cert.
p= 100,000−x
r= .045
t = 1

$(x)(.06)(1)$ + $(100,000−x)(.045)(1)$ =   5025
$.06x$ + $4500 − .045x$ =   5025

$.015x + 4500 = 5025$
$\quad\quad -4500 \quad -4500$

$$\frac{.015x}{.015} = \frac{525}{.015}$$

$x = 35,000$

$35,000 @ 6% rate
$65,000 @ 4.5% rate

Example 4

If Ben invests $4000 at 4% interest per year, how much additional money must he invest at 5.5% annual interest to ensure that the interest he receives each year is 4.5% of the total amount invested?

4% Rate   5.5% Rate   4.5% of Total Invested

$(4000)(.04)(1)$ + $(x)(.055)(1)$ = $.045(4000+x)$
160 + $.055x$ = $180 + .045x$
$\quad\quad - .045x$ $\quad - .045x$

$160 + .01x = 180$
$-160 \quad\quad -160$

$$\frac{.01x}{.01} = \frac{20}{.01}$$

$x = 2000$

$2000 at 5.5%

Example 5

Jack invests $1000 at a certain annual interest rate, and he invests another $2000 at an annual rate that is one-half percent higher. If he receives a total of $190 interest in one year, at what rate is the $1000 invested?

$\frac{1}{2}\%$ higher   .5%

$$\text{Investment 1} + \text{Investment 2} = 190$$
$$(1000)(x)(1) + (2000)(x+.005)(1) = 190$$
$$1000x + 2000x + 10 = 190$$
$$3000x + 10 = 190$$
$$\frac{3000x}{3000} = \frac{180}{3000}$$
$$x = .06$$

6% rate

Example 6

A woman earns 15% more than her husband. Together they make $69,875 per year. What is the husband's annual salary?

$$\text{husband earns} = x$$
$$\text{wife earns} = x + .15x = 1.15x$$

$$x + 1.15x = 69,875$$
$$\frac{2.15x}{2.15} = \frac{69,875}{2.15}$$
$$x = 32,500$$

$32,500 = husband's salary