1.7 Solving Absolute Value Equations and Inequalities

absolute value - the distance a number is from 0
- always positive because distance is positive

An absolute value equation is in the form $|ax + b| = c$.

To solve an absolute value equation when $c \geq 0$:

right → $ax + b = c$ or left → $ax + b = -c$

If $c < 0$, there is no solution.

Solve each absolute-value equation.

1. $|x - 2| = 5$ ← distance

$x - 2 = -5$ or $x - 2 = 5$
$+2 \quad +2 \qquad +2 \quad +2$
$x = -3$ or $x = 7$

2. $|x + 3| = -8$ ← distance

no solution b/c distance can't be negative

3. $|2x - 1| + 3 = 17$

isolate absolute value

$-3 \quad -3$

$|2x-1| = 14$

$2x-1 = -14$ or $2x-1 = 14$

$+1 \quad +1 \qquad +1 \quad +1$

$\frac{2x}{2} = -\frac{13}{2} \qquad \frac{2x}{2} = \frac{15}{2}$

$x = -\frac{13}{2}$ or $x = \frac{15}{2}$

4. $\frac{1}{2}|x + 5| - 6 = -3$

$+6 \quad +6$

$2 \cdot \frac{1}{2}|x+5| = 3 \cdot 2$

$|x+5| = 6$

$x+5 = -6$ or $x+5 = 6$

$-5 \quad -5 \qquad -5 \quad -5$

$x = -11$ or $x = 1$

Absolute Value Inequalities

$>$ $\geq$ } OR

$<$ $\leq$ } AND

OR

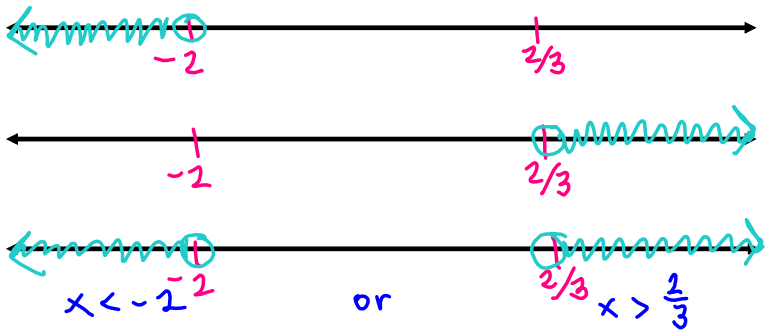
5. Solve $|3x + 2| > 4$.
Graph the solution on a number line.

$3x+2 < -4$ or $3x+2 > 4$

$-2 \quad -2$ $\quad$ $-2 \quad -2$

$\frac{3x}{3} < \frac{-6}{3}$ $\quad$ $\frac{3x}{3} > \frac{2}{3}$

$x < -2$ or $x > \frac{2}{3}$



AND

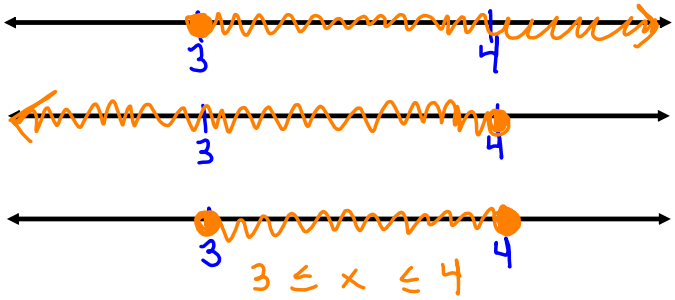
6. Solve $|2x - 7| \leq 1$.
Graph the solution on a number line.

$2x-7 \geq -1$ AND $2x-7 \leq 1$

$+7 \quad +7$ $\quad$ $+7 \quad +7$

$\frac{2x}{2} \geq \frac{6}{2}$ $\quad$ $\frac{2x}{2} \leq \frac{8}{2}$

$x \geq 3$ AND $x \leq 4$



OR

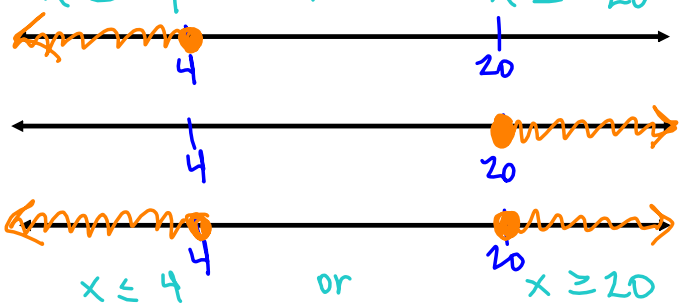
7. Solve $\left|\frac{1}{4}x - 3\right| \geq 2$.
Graph the solution on a number line.

$\frac{1}{4}x - 3 \leq -2$ or $\frac{1}{4}x - 3 \geq 2$

$+3 \quad +3$ $\quad +3 \quad +3$

$4 \cdot \frac{1}{4}x \leq 1 \cdot 4$ $\quad 4 \cdot \frac{1}{4}x \geq 5 \cdot 4$

$x \leq 4$ or $x \geq 20$



$x \leq 4$ or $x \geq 20$

AND

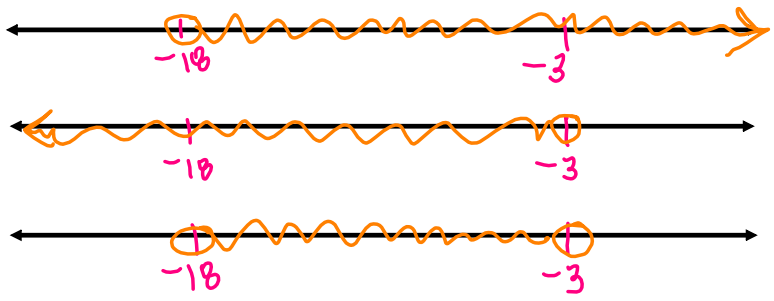
8. Solve $\left|\frac{2}{3}x + 7\right| < 5$.
Graph the solution on a number line.

$\frac{2}{3}x + 7 > -5$ AND $\frac{2}{3}x + 7 < 5$

$-7 \quad -7$ $\quad -7 \quad -7$

$\frac{3}{2} \cdot \frac{2}{3}x > -\frac{12}{1} \cdot \frac{3}{2}$ $\quad \frac{3}{2} \cdot \frac{2}{3}x < -\frac{2}{1} \cdot \frac{3}{2}$

$x > -18$ AND $x < -3$



$-18 < x < -3$