1.4 Rational Expressions

Rational Expression - the quotient of two algebraic expressions

REMEMBER: The denominator cannot equal 0!

An algebraic expression may not be defined for all values of the variable. The domain of an algebraic expression is the set of real numbers that the variable is permitted to have.

$x+4 \neq 0$

For example, in the function $f(x) = \dfrac{1}{x+4}$

x cannot equal -4. So the domain is $x \neq -4$.

$(-\infty,-4) \cup (-4,\infty)$

Example 1: Find the domain of each function.

a) $f(x) = \dfrac{1}{x^2 - x}$

$x^2 - x \neq 0$

$x(x-1) \neq 0$

$x \neq 0$

$x - 1 \neq 0$
$+1 \quad +1$
$x \neq 1$

$(-\infty,0) \cup (0,1) \cup (1,\infty)$

b) $g(x) = \sqrt{9 - x^2}$

can't be negative

positive

$9 - x^2 \geq 0$

$(3-x)(3+x) \geq 0$

$3 - x = 0 \qquad 3 + x = 0$

$x = 3 \qquad x = -3$

$- \quad + \quad -$

$-4 \quad -3 \quad 0 \quad 3 \quad 4$

$(3--4)(3+-4)$, $(7)(-1)$, -7

$(3-0)(3+0)$, $(3)(3)$, 9

$(3-4)(3+4)$, $(-1)(7)$, -7

$\boxed{D: [-3,3]}$

c) $h(t) = \dfrac{1}{\sqrt{t+1}}$

can't equal 0 & can't be negative

can be positive

$t + 1 > 0$
$-1 \quad -1$
$t > -1$

$\boxed{D: (-1,\infty)}$

Simplifying Rational Expressions

Factor both numerator and denominator and cancel common factors.

Example 2: Simplify $\dfrac{x^2 - 1}{x^2 + x - 2}$

s1 p−2

2 −1

1

$$\frac{\cancel{(x-1)}(x+1)}{(x+2)\cancel{(x-1)}}$$

$$\boxed{\frac{x+1}{x+2}}$$

Multiplying Fractional Expressions

Factor all numerators and denominators, then cancel common factors and multiply.

Example 3: Perform the indicated operation.

$$\frac{x^2 + 2x - 3}{x^2 + 8x + 16} \cdot \frac{3x + 12}{x - 1}$$

$$\frac{\cancel{(x-1)}(x+3)}{(x+4)\cancel{(x+4)}} \cdot \frac{3\cancel{(x+4)}}{\cancel{x-1}} = \boxed{\frac{3(x+3)}{x+4}}$$

Dividing Fractional Expressions

Multiply by the reciprocal (flip the fraction after the division sign).

Example 4: Perform the indicated operation.

$$\frac{x-4}{x^2-4} \div \frac{x^2-3x-4}{x^2+5x+6}$$

$$\frac{x-4}{x^2-4} \cdot \frac{x^2+5x+6}{x^2-3x-4}$$

$$\frac{\cancel{x-4}}{\cancel{(x+2)}(x-2)} \cdot \frac{(x+3)\cancel{(x+2)}}{\cancel{(x-4)}(x+1)} = \boxed{\frac{x+3}{(x-2)(x+1)}}$$

Adding & Subtracting Fractional Expressions

You MUST have common denominators.

Example 5: Perform the indicated operation.

a) LCD: $(x-1)(x+2)$

$$\frac{3}{x-1} + \frac{x}{x+2}$$

$$\frac{3(x+2)}{(x-1)(x+2)} + \frac{x(x-1)}{(x+2)(x-1)}$$

$$\frac{3x+6}{(x-1)(x+2)} + \frac{x^2-x}{(x-1)(x+2)}$$

$$\boxed{\frac{x^2+2x+6}{(x-1)(x+2)}}$$

b) LCD: $(x+1)(x+1)(x-1)$

$$\frac{1}{x^2-1} - \frac{2}{(x+1)^2}$$

$(x-1)(x+1)$ $(x+1)(x+1)$

$$\frac{1(x+1)}{(x+1)(x+1)(x-1)} - \frac{2(x-1)}{(x+1)(x+1)(x-1)}$$

$$\frac{x+1}{(x+1)(x+1)(x-1)} + \frac{-2x+2}{(x+1)(x+1)(x-1)}$$

$$\boxed{\frac{-x+3}{(x+1)(x+1)(x-1)}}$$

Compound Fraction

A fraction in which the numerator and/or the denominator is also a fractional expression.

Example 6: Perform the indicated operation.

$$\frac{\frac{x}{y}+\frac{1\cdot y}{1\cdot y}}{\frac{1\cdot x}{1\cdot x}-\frac{y}{x}} \rightarrow \frac{\frac{x}{y}+\frac{y}{y}}{\frac{x}{x}-\frac{y}{x}} \rightarrow \frac{\frac{x+y}{y}}{\frac{x-y}{x}}$$

$$\frac{x+y}{y} \div \frac{x-y}{x}$$

$$\frac{x+y}{y} \cdot \frac{x}{x-y}$$

$$\boxed{\frac{x(x+y)}{y(x-y)}}$$

Example 7: Perform the indicated operation.

LCD: $a(a+h)$

$$\frac{\frac{1\cdot a}{a\cdot(a+h)}-\frac{1\,(a+h)}{(a)(a+h)}}{h} \rightarrow \frac{\frac{a}{a(a+h)}+\frac{-a-h}{a(a+h)}}{h} \rightarrow \frac{\frac{-h}{a(a+h)}}{h}$$

$$\frac{-h}{a(a+h)} \div \frac{h}{1}$$

$$\frac{-\cancel{h}}{a(a+h)} \cdot \frac{1}{\cancel{h}}$$

$$\boxed{\frac{-1}{a(a+h)}}$$

Example 8: Perform the indicated operation.

$$\frac{(1 + x^2)^{1/2} - x^2(1 + x^2)^{-1/2}}{1 + x^2}$$

$$\frac{(1+x^2)^{-1/2}\left[(1+\cancel{x^2}) - \cancel{x^2}\right]}{1+x^2} = \frac{(1+x^2)^{-1/2}}{(1+x^2)^1} = \boxed{\frac{1}{(1+x^2)^{3/2}}}$$

Rationalizing the Denominator or the Numerator

If a fraction has a denominator of the form $a + b\sqrt{c}$, we rationalize the denominator by multiplying both numerator and denominator by the conjugate radical, which is the same radical with the opposite sign.

$5+\sqrt{3}$

$5-\sqrt{3}$

Example 9: Rationalize the denominator.

FOIL $\quad \frac{1}{1+\sqrt{2}} \cdot \frac{1-\sqrt{2}}{1-\sqrt{2}}$

$$\frac{1-\sqrt{2}}{1-\cancel{\sqrt{2}}+\cancel{\sqrt{2}}-2} = \frac{1-\sqrt{2}}{-1}$$

$$\boxed{-1+\sqrt{2}}$$