2.2 Properties of Exponents

KEY CONCEPT *For Your Notebook*

Properties of Exponents

Let a and b be real numbers and let m and n be integers.

Property Name	Definition	Example
Product of Powers	$a^m \cdot a^n = a^{m+n}$	$5^3 \cdot 5^{-1} = 5^{3+(-1)} = 5^2 = 25$
Power of a Power	$(a^m)^n = a^{mn}$	$(3^3)^2 = 3^{3 \cdot 2} = 3^6 = 729$
Power of a Product	$(ab)^m = a^m b^m$	$(2 \cdot 3)^4 = 2^4 \cdot 3^4 = 1296$
Negative Exponent	$a^{-m} = \frac{1}{a^m}, a \neq 0$	$7^{-2} = \frac{1}{7^2} = \frac{1}{49}$
Zero Exponent	$a^0 = 1, a \neq 0$	$(-89)^0 = 1$
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$	$\frac{6^{-3}}{6^{-6}} = 6^{-3-(-6)} = 6^3 = 216$
Power of a Quotient	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$	$\left(\frac{4}{7}\right)^2 = \frac{4^2}{7^2} = \frac{16}{49}$

Simplify.

1. $(a^2b^2c^3)(a^2bc)$

$a^4 b^3 c^4$

2. $(-2r^2s)^3$

$(-2)^3 (r^{2 \cdot 3})(s)^3$

$-8r^6 s^3$

3. $(-xy^2)^3(-2x^2y)^2(-4xy)$

$(-x)^3(y^{2\cdot 3})(-2)^2(x^{2\cdot 2})(y)^2(-4xy)$

$-x^3y^6 \cdot 4x^4y^2 \cdot -4xy$

$16x^8y^9$

Simplify.

4. $\frac{-24x^2y^2}{6xy^4}$

$\frac{-4xy^{-2}}{1}$

$\frac{-4x}{y^2}$

5. $\left(\frac{-6y^5}{3y^6}\right)^2$

$(-2y^{-1})^2$

$\left(\frac{-2}{y^1}\right)^2$

$\frac{(-2)^2}{(y)^2}$

$\frac{4}{y^2}$

6. $\frac{(a^2b^3)^0}{4}$

$\frac{1}{4}$

Simplify.

7. $(4x^{-3}y^2)^{-3}$

$4^{-3}(x^{-3 \cdot -3})(y^{2 \cdot -3})$

$\frac{1}{4^3} \cdot \frac{x^9}{1} \cdot \frac{y^{-6}}{1}$

$\frac{x^9}{4^3 y^6}$

$\frac{x^9}{64 y^6}$

8. $\left(\frac{3g^2h^{-1}}{2g^{-5}h^3}\right)^{-4}$

change to + FLIP

$\left(\frac{2g^{-5}h^3}{3g^2h^{-1}}\right)^4$

$\left(\frac{2}{3}g^{-7}h^4\right)^4$

$\left(\frac{2h^4}{3g^7}\right)^4$

$\frac{16h^{16}}{81g^{28}}$

9. $\left[\frac{(-k^5m^{-2})^3}{(k^3m^{-7})^{-6}}\right]^{-2}$

$\frac{(-k^5m^{-2})^{-6}}{(k^3m^{-7})^{12}}$

$\frac{+k^{-30}m^{12}}{k^{36}m^{-84}}$

$k^{-66}m^{96}$

$\frac{m^{96}}{k^{66}}$

Simplify.

10. $\dfrac{(3c^{-2}d^{3})(5cd^{-8})}{(c^{-3})^{4}d^{-2}}$

$\dfrac{15c^{-1}d^{-5}}{c^{-12}d^{-2}}$

$15c^{11}d^{-3}$

$\dfrac{15c^{11}}{d^{3}}$

11. $\left(\dfrac{p^{-3}}{4r}\right)^{-3}\left(\dfrac{5r}{p^{-7}}\right)^{-2}$

$\left(\dfrac{4r}{p^{-3}}\right)^{3}\left(\dfrac{p^{-7}}{5r}\right)^{2}$

$\dfrac{64r^{3}}{p^{-9}}\cdot\dfrac{p^{-14}}{25r^{2}}$

$\dfrac{64p^{-14}r^{3}}{25p^{-9}r^{2}}$

$\dfrac{64}{25}p^{-5}r$

$\dfrac{64r}{25p^{5}}$

Rational Exponents

A radical expression contains a radical symbol and index number or a rational (fraction) exponent.

Radical Notation

index

$\sqrt[2]{32x^5}$ square root — The only one without an index number.

$\sqrt[3]{64y^6}$ cube root

$\sqrt[4]{16x^8}$ fourth root

$\sqrt[5]{64}$ fifth root

Exponential Notation

a. $\sqrt[n]{a} = a^{\frac{1}{n}}$ (index number becomes the denominator on the exponent)

b. $\sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m = a^{\frac{m}{n}}$ (exponent becomes the numerator)

c. $a^{-\frac{m}{n}} = \frac{1}{\sqrt[n]{a^m}}$

Simplify.

$1024 = 2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2$

12. $16^{\frac{1}{4}}$

$\left(\sqrt[4]{16}\right)^1$

$(2)^1$

2

13. $27^{\frac{4}{3}}$

$\left(\sqrt[3]{27}\right)^4$

$(3)^4$

81

14. $1024^{-\frac{2}{5}}$

$\frac{1}{\left(\sqrt[5]{1024}\right)^2}$

$\frac{1}{(4)^2}$

$\frac{1}{16}$

15. $64^{\frac{5}{6}}$ $2\cdot2\cdot2\cdot2\cdot2\cdot2$

$\left(\sqrt[6]{64}\right)^5$

$(2)^5$

32

16. $36^{\frac{3}{2}}$

$\left(\sqrt[2]{36}\right)^3$

$(6)^3$

216

17. $216^{-\frac{2}{3}}$

$\frac{1}{\left(\sqrt[3]{216}\right)^2}$

$\frac{1}{(6)^2}$

$\frac{1}{36}$