1.5 Segment and Angle Bisectors

**midpoint** - the point in the middle of a segment
- it divides (or **bisects**) the segment into two congruent segments

**segment bisector** - a segment, ray, line, or plane that intersects a segment at its midpoint

Example 1: K is the midpoint of $FG$. Find $FK$ and $KG$.

Example 2: M is the midpoint of $JL$. Find $ML$ and $JL$. 

![Diagram showing midpoints and bisectors](https://via.placeholder.com/150)
Example 3: \( y \) is a segment bisector of \( FH \). Find the value of \( x \).

\[
9x = 36
\]

\[
x = 4
\]

Midpoint Formula

To find the coordinates of the midpoint of a segment in a coordinate plane you take the average of the \( x \)-coordinates and the \( y \)-coordinates.

To find the midpoint of a segment with endpoints \( A(x_1, y_1) \) and \( B(x_2, y_2) \):

\[
M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)
\]
Example 4: Find the coordinates of the midpoint of $\overline{AB}$ with the endpoints $A(-2, 3)$ and $B(5, -2)$.

\[
\left(\frac{-2+5}{2}, \frac{3-2}{2}\right) \rightarrow \left(\frac{3}{2}, \frac{1}{2}\right)
\]

Example 5: Find the coordinates of the midpoint of $\overline{DE}$ with the endpoints $D(3, 5)$ and $E(-4, 0)$.

\[
\left(\frac{3-4}{2}, \frac{5+0}{2}\right) \rightarrow \left(-\frac{1}{2}, \frac{5}{2}\right)
\]

Example 6: The midpoint of $\overline{RP}$ is $M(2,4)$. One endpoint is $R(-1,7)$. Find the coordinates of $P$.

\[
2 \cdot \frac{-1+x}{2} = 2 \cdot 2 \quad 2 \cdot \frac{7+y}{2} = 4 \cdot 2
\]

\[
-1 + x = 4 \quad \frac{7+y}{2} = 4
\]

\[
+1 +1
\]

\[
x = 5 \quad y = 1
\]

$P(5, 1)$
Example 7: The midpoint of \( XY \) is \( M(3, -4) \). One endpoint is \( Y(-3, -1) \). Find the coordinates of \( X \).

\[
\begin{align*}
2 \cdot \frac{x - 3}{2} &= 3 \cdot 2 \\
2 \cdot \frac{y - 1}{2} &= -4 \cdot 2
\end{align*}
\]

\[
\begin{align*}
x - 3 &= 6 \\
+3 &+3 \\
x &= 9
\end{align*}
\]

\[
\begin{align*}
y - 1 &= -8 \\
+1 &+1 \\
y &= -7
\end{align*}
\]

\( X(9, -7) \)

**angle bisector** - a ray that divides an angle into two congruent adjacent angles

\[
\overline{CD} \text{ is a bisector of } \angle ACB
\]

\[
m \angle ACD = m \angle BCD
\]

\[
\angle ACD \equiv \angle BCD
\]
**Example 8:** The ray $\overrightarrow{FH}$ bisects the angle $\angle EFG$. Given that $m\angle EFG = 120^\circ$, what are the measures of $\angle EFH$ and $\angle HFG$?

![Diagram of Example 8]

- $m\angle EFH = 60^\circ$
- $m\angle HFG = 60^\circ$

**Example 9:** $\overrightarrow{RQ}$ bisects $\angle PRS$. The measures of the two congruent angles are $(x + 40)^\circ$ and $(3x - 20)^\circ$. Solve for $x$ and find the measure of each angle.

![Diagram of Example 9]

- $m\angle SRQ = 70^\circ$
- $m\angle PQR = 70^\circ$

Solve for $x$:

\[
\begin{align*}
3x - 20 &= x + 40 \\
-2x &\quad -2x \\
2x &= 60 \\
\frac{2x}{2} &= \frac{60}{2} \\
x &= 30
\end{align*}
\]