

Section 1.3 CONTINUED

Factoring Expressions with Fractional Exponents

Factor out the power with the *smallest* exponent.

Example 1

Factor $3x^{3/2} - 9x^{1/2} + 6x^{-1/2}$.

$$3x^{-1/2} (x^2 - 3x + 2)$$

$$3x^{-1/2} (x-2)(x-1)$$

$$\begin{array}{r} \text{sum } -3 \text{ prod } 2 \\ \hline \frac{-2}{1} \quad \frac{-1}{1} \end{array}$$

Example 2

Factor $x^{9/2} - x^{1/2}$.

$$x^{1/2} \left(\frac{x^4}{(x^2)^2} - \frac{1}{(1)^2} \right)$$

$$x^{1/2} \left(\frac{x^2 - 1}{(x)^2} \frac{(1)^2}{(1)^2} \right) (x^2 + 1)$$

$$x^{1/2} (x-1)(x+1)(x^2+1)$$

Example 3

Factor $10x^{-1/2} + 7x^{1/2} + x^{3/2}$.

$$x^{-1/2} (10 + 7x + x^2)$$

$$x^{-1/2} (2+x)(5+x)$$

Sum 7 prod 10

$\frac{5}{10}$	$\frac{2}{10}$
↓	↓
$\frac{1}{2}$	$\frac{1}{5}$

Example 4

Factor $(2+x)^{-2/3}x + (2+x)^{1/3}$.

$$(2+x)^{-2/3} [x + (2+x)]$$

$$(2+x)^{-2/3} [x + 2+x]$$

$$(2+x)^{-2/3} (2+2x)$$

$$2(2+x)^{-2/3} (1+x)$$

Example 5Factor $(x - 4)^{3/2} - (x - 4)^{7/2}$.

$$(x-4)^{3/2} \left[1 - (x-4)^2 \right]$$

$$(x-4)^{3/2} \left[1 - (x^2 - 8x + 16) \right]$$

$$(x-4)^{3/2} \left[1 - x^2 + 8x - 16 \right]$$

$$(x-4)^{3/2} (-x^2 + 8x - 15) \quad \begin{array}{l} \text{sum } 8 \text{ prod } 15 \\ \frac{-5}{1} \quad \frac{-3}{1} \end{array}$$

$$- (x-4)^{3/2} (x^2 - 8x + 15)$$

$$- (x-4)^{3/2} (x-5)(x-3)$$

Example 6Factor $(x^2 + 5)^{5/2} - 3(x^2 + 5)^{1/2}$.

$$(x^2+5)^{1/2} \left[\frac{(x^2+5)(x^2+5)}{(x^2+5)^2} - 3 \right]$$

$$(x^2+5)^{1/2} \left[x^4 + 10x^2 + 25 - 3 \right]$$

$$(x^2+5)^{1/2} (x^4 + 10x^2 + 22)$$

Example 7Factor $x^{-1/2}(x-2)^{3/2} + x^{1/2}(x-2)^{1/2}$.

$$x^{-1/2}(x-2)^{1/2} \left[(x-2)^1 + x \right]$$

$$x^{-1/2}(x-2)^{1/2} (2x-2)$$

$$2x^{-1/2}(x-2)^{1/2} (x-1)$$

Example 8

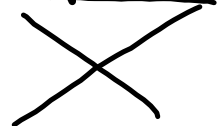
Factor completely.

$$\cdot \underline{3}(x^2-5)^4(\underline{2x})(x+3)^5 + (x^2-5)^5(\underline{8})(x+3)^4$$

$$2(x^2-5)^4(x+3)^4 \left[\overset{\curvearrowright}{3x(x+3)} + \overset{\curvearrowright}{4(x^2-5)} \right]$$

$$2(x^2-5)^4(x+3)^4 \left[3x^2 + 9x + 4x^2 - 20 \right]$$

$$2(x^2-5)^4(x+3)^4 (7x^2 + 9x - 20)$$

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Example 9

Factor completely.

$$2(3x-2)^2(4)(x+1)^{-1/2} + (3x-2)^3\left(\frac{1}{2}\right)(x+1)^{-3/2}$$

$$\frac{1}{2}(3x-2)^2(x+1)^{-3/2} \left[16x+16 + (3x-2) \right]$$

$$\frac{1}{2}(3x-2)^2(x+1)^{-3/2} (19x+14)$$

Example 10

Simplify.

$$\frac{3(x+2)^2(x-3)^2 - (x+2)^3(2)(x-3)}{(x-3)^4}$$

$$\frac{(x+2)^2(x-3) [3(x-3) - 2(x+2)]}{(x-3)^4}$$

$$\frac{(x+2)^2(x-3) [3x-9-2x-4]}{(x-3)^4}$$

$$\frac{(x+2)^2 \cancel{(x-3)} (x-13)}{(x-3)^{\cancel{4}+3}}$$

$$\frac{(x+2)^2(x-13)}{(x-3)^3}$$

Example 11

Simplify.

$$\frac{(1+x^2)^{1/2} - x^2(1+x^2)^{-1/2}}{1+x^2}$$

$$\frac{(1+x^2)^{-1/2} [(1+x^2) - x^2]}{1+x^2}$$

$$\frac{(1+x^2)^{-1/2}}{(1+x^2)^1}$$

$$\frac{1}{(1+x^2)^{3/2}}$$