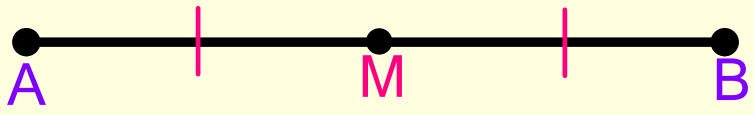
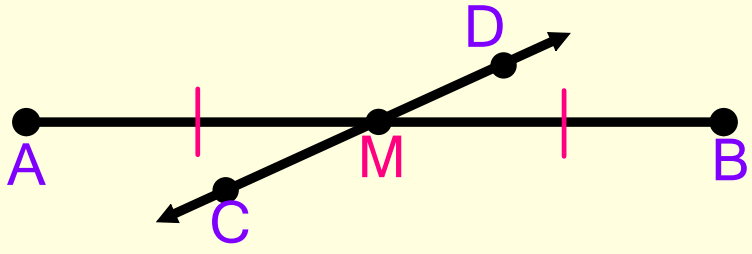
1.3 Use Midpoint & Distance Formulas

The midpoint of a segment is the point that divides the segment into two congruent segments.



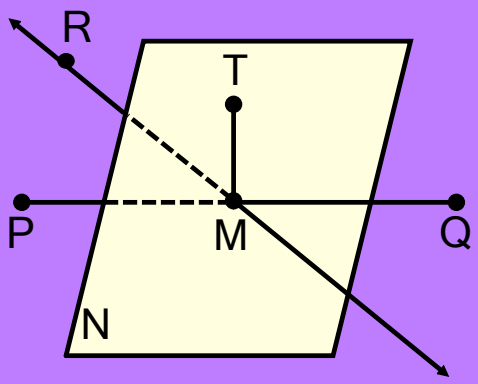
M is the midpoint of $\overline{AB}$.
So AM = MB and $\overline{AM} \cong \overline{MB}$.

A segment bisector is a point, ray, line, line segment, or plane that intersects the segment at its midpoint.



$\overline{CD}$ is the segment bisector of $\overline{AB}$.
So AM = MB and $\overline{AM} \cong \overline{MB}$.

What are the segment bisectors of $\overline{PQ}$?



$\overline{TM}$

$\overleftrightarrow{RM}$

plane N

Example 1

If Y is the midpoint of $\overline{XZ}$,
XY = 2a + 11 and YZ = 4a - 5,
find the value of a and the measure of $\overline{XZ}$.

$2a+11 = 4a-5$

$-4a \quad -4a$

$-2a+11 = -5$

$-11 \quad -11$

$\frac{-2a}{-2} = \frac{-16}{-2}$

$a = 8$

$XY + YZ = XZ$

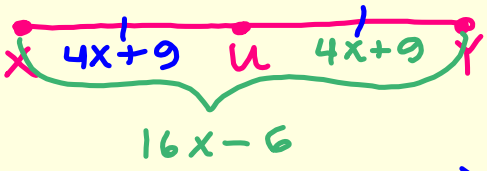
$2(8)+11+4(8)-5 = XZ$

$16+11+32-5 = XZ$

$54 = XZ$

Example 2

If U is the midpoint of $\overline{XY}$,
XY = 16x - 6 and UY = 4x + 9,
find the value of x and the measure of $\overline{XY}$.



$2(4x+9) = 16x-6$

$(4x+9)+(4x+9) = 16x-6$

$8x+18 = 16x-6$

$-16x \quad -16x$

$-8x+18 = -6$

$-18 \quad -18$

$-8x = -24$

$\frac{-8x}{-8} = \frac{-24}{-8}$

$x = 3$

$XY = 16(3)-6$

$XY = 48-6$

$XY = 42$

MIDPOINT FORMULAS

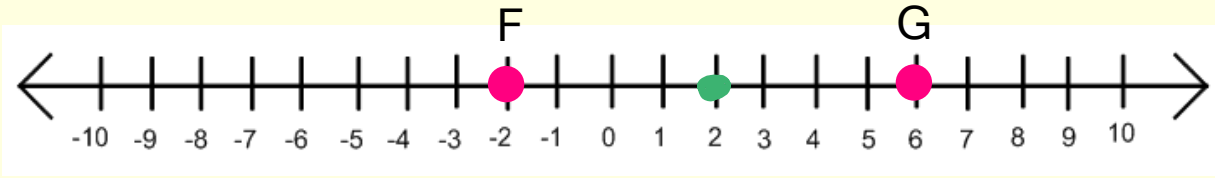
1. On a number line, the coordinate of the midpoint of a segment whose endpoints have coordinates a and b is $\frac{a+b}{2}$.

2. In a coordinate plane, the coordinates of the midpoint of a segment whose endpoints have coordinates (x_1, y_1) and (x_2, y_2) are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

Example 3

Use the number line to find the coordinate of the midpoint of $\overline{FG}$.

$$\frac{-2+6}{2} = \frac{4}{2} = 2$$



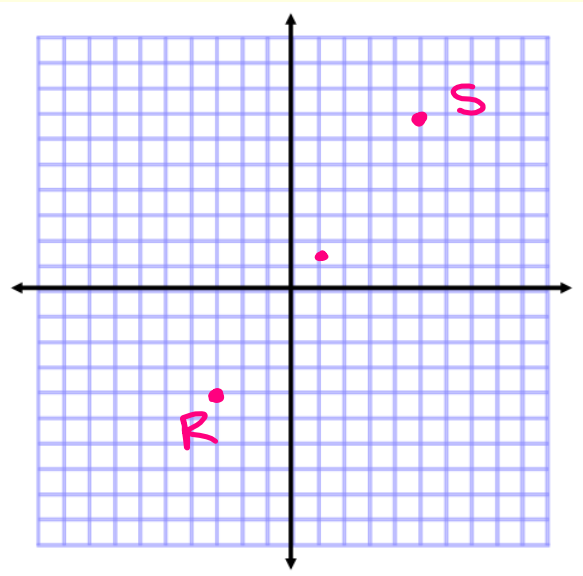
Example 4

If the coordinate of H is -13 and the coordinate of J is 10, what is the coordinate of the midpoint of $\overline{HJ}$?

$$\frac{-13+10}{2} = \frac{-3}{2} \text{ or } -1.5$$

Example 5

Find the coordinates of Q, the midpoint of $\overline{RS}$, if the endpoints of RS are R(-3, -4) and S(5, 7).



$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\left(\frac{-3+5}{2} \quad \frac{-4+7}{2}\right)$$

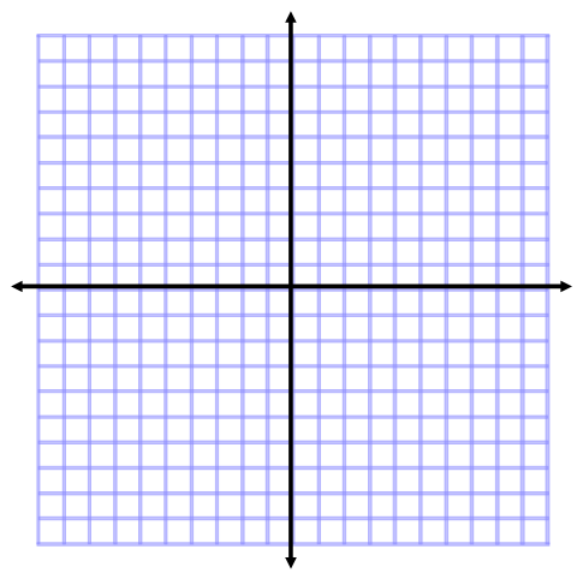
$$\left(\frac{2}{2} \quad \frac{3}{2}\right)$$

$$(1, 1.5)$$

Example 6

Find the coordinates of the midpoint of $\overline{VW}$ for V(3, -6) and W(7, 2).

$x_1\ y_1 \qquad x_2\ y_2$



$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$\left(\frac{3+7}{2} \quad \frac{-6+2}{2}\right)$$

$$\left(\frac{10}{2}, \frac{-4}{2}\right)$$

$$(5, -2)$$

Example 7

Find the coordinates of point Q if L(4, -6) is the midpoint of $\overline{NQ}$ and the coordinates of N are (8, -9).

N (8, -9) L (4, -6) Q (x, y)

$$2 \cdot \frac{8+x}{2} = 4 \cdot 2 \qquad 2 \cdot \frac{-9+y}{2} = -6 \cdot 2$$

$$8 + x = 8 \qquad -9 + y = -12$$

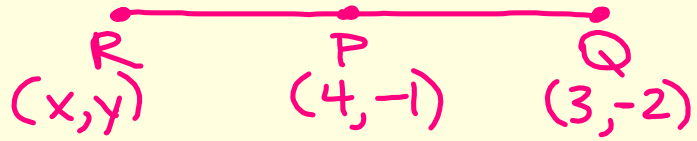
$$-8 \quad -8 \qquad +9 \quad +9$$

$$x = 0 \qquad y = -3$$

$$Q(0, -3)$$

Example 8

The midpoint of $\overline{RQ}$ is P(4, -1). What are the coordinates of R if Q is at (3, -2)?



$$2 \cdot \frac{x+3}{2} = 4 \cdot 2 \qquad 2 \cdot \frac{y+(-2)}{2} = -1 \cdot 2$$

$$x + 3 = 8 \qquad y - 2 = -2$$
$$-3 \quad -3 \qquad +2 \quad +2$$
$$x = 5 \qquad y = 0$$

$R(5, 0)$

DISTANCE FORMULA

The distance d between any two points with coordinates (x_1, y_1) and (x_2, y_2) is given by the formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example 9

Find PQ for P(-3, -5) and Q(4, -6).

$x_1\ y_1 \qquad x_2\ y_2$

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$\sqrt{(4+3)^2+(-6+5)^2}$$

$$\sqrt{(7)^2+(-1)^2}$$

$$\sqrt{49+1}$$

$$\sqrt{50}=\sqrt{2\cdot 5\cdot 5}$$

$$PQ=5\sqrt{2}$$

2	50
5	25
	5

Example 10

Find JK for J(9, -5) and K(-6, 12).

$x_1\ y_1 \qquad x_2\ y_2$

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$\sqrt{(-6-9)^2+(12+5)^2}$$

$$\sqrt{(-15)^2+(17)^2}$$

$$\sqrt{225+289}$$

$$\sqrt{514}$$

2	514
	257