1.3 ALGEBRAIC EXPRESSIONS

variable- a letter that represents a number

constant- a fixed (or specific) number

domain of a variable- set of numbers that the variable is allowed to have

examples: $\sqrt{x}$ has a domain of $x \geq 0$ or $[0,\infty)$ (index is even, cannot be negative)

$\frac{2}{x-3}$ has a domain of $x \neq 3$ or $(-\infty,3) \cup (3,\infty)$ ($x-3 \neq 0$, $+3\ +3$, $x \neq 3$; cannot be zero)

$\sqrt[3]{x}$ has a domain of all real numbers or $(-\infty,\infty)$ (index is odd)

polynomials- simple algebraic expressions that include addition, subtraction, and multiplication

degree of a polynomial- highest power (exponent) of the variable

degree	degree name
1	linear
2	quadratic
3	cubic
4	quartic
5	quintic

monomial- one term

binomial- two terms

trinomial- three terms

standard form- descending exponential order

Write each sum or difference as a polynomial in standard form. Then classify the polynomial by degree and by number of terms.

1. $(x^3 + x^2 + x + 1) + (2x^3 + 3x^2 + x + 1)$

$3x^3 + 4x^2 + 2x + 2$

cubic polynomial

2. $(1 - 5x + x^3) + (2x^4 + 5x^3 + 10x^2)$

$-2x^4 - 4x^3 + 10x^2 - 5x + 1$

quartic polynomial

Write each sum or difference as a polynomial in standard form. Then classify the polynomial by degree and by number of terms.

3. $(\frac{2}{3}x + \frac{2}{3}x^3 + 1) + (\frac{2}{3} + \frac{1}{3}x^2 + \frac{1}{3}x)$

$\frac{2}{3}x^3 - \frac{1}{3}x^2 + \frac{1}{3}x + \frac{1}{3}$

cubic polynomial

4. $(x^4 + 5x^2 + x) - (x^4 + 2x^3 + x + 4)$

$-2x^3 + 5x^2 + 4$

cubic trinomial

Multiply.

5. $(2x + 1)(3x - 5)$

$6x^2 - 10x + 3x - 5$

$6x^2 - 7x - 5$

6. $(x^2 - 3)(x^3 + 2x + 1)$

$x^2(x^3 + 2x + 1) - 3(x^3 + 2x + 1)$

$x^5 + 2x^3 + x^2 - 3x^3 - 6x - 3$

$x^5 - x^3 + x^2 - 6x - 3$

7. $(1 + \sqrt{x})(2 - 3\sqrt{x})$

$\sqrt{x} \cdot \sqrt{x}$

$(1)(2) + (1)(-3\sqrt{x}) + (\sqrt{x})(2) + (\sqrt{x})(-3\sqrt{x})$

$2 \quad -3\sqrt{x} \quad +2\sqrt{x} \quad -3x$

$2 \quad -\sqrt{x} \quad -3x$

SPECIAL PRODUCTS

diff of squares

$(a - b)(a + b) = a^2 - b^2$

$(a + b)^2 = a^2 + 2ab + b^2$

$(a - b)^2 = a^2 - 2ab + b^2$

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

Multiply.

8. $(2x - \sqrt{y})(2x + \sqrt{y})$

$(2x)^2 - (\sqrt{y})^2$

$4x^2 - y$

9. $(\overset{a}{3x} + \overset{b}{5})^2$ $a^2 + 2ab + b^2$

$(3x)^2 + 2(3x)(5) + (5)^2$

$9x^2 + 30x + 25$

10. $(\underbrace{2x^2}_{a} - \underbrace{4}_{b})^3$ $a^3 - 3a^2b + 3ab^2 - b^3$

$(2x^2)^3 - 3(2x^2)^2(4) + 3(2x^2)(4)^2 - (4)^3$

$8x^6 - 3(4x^4)(4) + 3(2x^2)(16) - 64$

$8x^6 - 48x^4 + 96x^2 - 64$

FORMULAS/PATTERNS TO KNOW

$x^2 - y^2 = (x-y)(x+y)$

$x^2 + 2xy + y^2 = (x+y)^2$

$x^2 - 2xy + y^2 = (x-y)^2$

* $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$ ← do not factor

sum of cubes

* $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$ ← do not factor

diff. of cubes

STEPS TO FACTOR POLYNOMIALS

Step 1: Factor out any GCF.

Step 2: For a binomial, check if it is the difference of squares. sum of cubes, diff. of cubes.

Step 3: For a trinomial, check to see if it matches any pattern. If not, jump into a sum/product chart.

Step 4: For 4 terms, use the grouping method.

Step 5: See if any factors can be factored further.

FACTORING QUADRATIC TRINOMIALS

Example: $5x^2 + 17x + 14$ (product, sum, LC)

sum 17 product 70

$\frac{10}{5}$ $\frac{7}{5}$

$2 \downarrow$ LC 1 $\frac{7}{5}$

1. Make a sum/product chart.
2. Divide each number by the leading coefficient.
3. Reduce each fraction if possible.
4. Denominator = coefficient for first term (variable)
   Numerator = constant or coefficient of last term

$(x+2)(5x+7)$

FACTOR EACH POLYNOMIAL.

11. $3x^2 - 6x$

$3x(x-2)$

12. $8x^4y^2 + 6x^3y^3 - 2xy^4$

$2xy^2(4x^3 + 3x^2y - y^2)$

FACTOR EACH POLYNOMIAL.

13. $x^2 + 7x + 12$

sum 7 product 12

$4 \quad 3$

$1 \quad 1$

$(x+4)(x+3)$

14. $6x^2 + 7x - 5$

sum 7 product -30

$-3 \quad 10$

$6 \quad 6$

$-1 \quad 5$

$2 \quad 3$

$(2x-1)(3x+5)$

FACTOR EACH POLYNOMIAL.

15. $4x^2 - 25$

$(2x)^2 \quad (5)^2$

$(2x-5)(2x+5)$

16. $27x^3 - 1$

$(3x)^3 \quad (1)^3$

$(x-y)(x^2+xy+y^2)$

$(3x-1)((3x)^2+(3x)(1)+(1)^2)$

$(3x-1)(9x^2+3x+1)$

FACTOR EACH POLYNOMIAL.

17. $x^6 - 8$

$(x^2)^3 \quad (2)^3$

$(x-y)(x^2+xy+y^2)$

$(x^2-2)((x^2)^2+(x^2)(2)+(2)^2)$

$(x^2-2)(x^4+2x^2+4)$

18. $x^6 - 16$

$(x^3)^2 \quad (4)^2$

$(x-y)(x+y)$

$(x^3-4)(x^3+4)$

FACTOR EACH POLYNOMIAL.

19. $x^2 + 6x + 9$

sum 6 prod 9

3 3

1 1

$(x+3)(x+3)$

or

$(x+3)^2$

20. $4x^2 - 4xy + y^2$

sum -4 product 4

-2 -2

4 4

$-\frac{1}{2}$ $-\frac{1}{2}$

$(2x-y)(2x-y)$

or

$(2x-y)^2$

FACTOR EACH POLYNOMIAL.

21. $2x^4 - 8x^2$

$2x^2(x^2-4)$

$(x)^2 \quad (2)^2$

$2x^2(x-2)(x+2)$

22. $x^5y^2 - xy^6$

$xy^2(x^4 - y^4)$

$(x^2)^2 \quad (y^2)^2$

$xy^2(x^2-y^2)(x^2+y^2)$

$(x)^2 \quad (y)^2$

$xy^2(x-y)(x+y)(x^2+y^2)$

FACTOR EACH POLYNOMIAL.

23. $(ra + rb) + (sa + sb)$

$r(a+b) + s(a+b)$

$(a+b)(r+s)$

24. $(x^2 - 12x) + (3x - 36)$

$x(x-12) + 3(x-12)$

$(x-12)(x+3)$

FACTOR EACH POLYNOMIAL.

25. $(6cd^2 - 8cd) - (9d + 12)$

$2cd(3d-4) - 3(3d-4)$

$(3d-4)(2cd-3)$

26. $(2x^2y - x) - (3 + 6xy)$

$x(2xy-1) - 3(1+2xy)$

$x(2xy-1) + 3(2xy+1)$

$(2xy-1)(x+3)$

FACTOR EACH POLYNOMIAL.

27. Let $w = x-2$

$(x - 2)^2 - 14(x - 2) + 48$

$w^2 - 14w + 48$

Sum -14 prod 48

-8 -6

$(w-8)(w-6)$

$(x-2-8)(x-2-6)$

$(x-10)(x-8)$

28. $w = z+4$

$(z + 4)^2 + 6(z + 4) - 16$

$w^2 + 6w - 16$

Sum 6 prod -16

8 -2

$(w+8)(w-2)$

$(z+4+8)(z+4-2)$

$(z+12)(z+2)$