1.3 Segments and Their Measures

In geometry, rules that are accepted without proof are called postulates.

**Postulate 1: Ruler Postulate**

The points on a line can be matched with a real number. The real number that corresponds to a point is the *coordinate* of the point.

The **distance** between points A and B, written as AB, is the absolute value of the difference between the coordinates.

AB is also called the *length* of AB.

\[ |4 - (-3)| = |7| = 7 \quad |3 - 4| = |-7| = 7 \]
Example 1: What is the length of \( XY \)?

\[ XY = 3 \frac{1}{2} \]

When 3 points lie on a line, one of them is between the other two.

B is between A and C.
Example 2
Find AB, BC, and AC on the number line shown below.

\[ AB = 1 \frac{1}{2} \]
\[ BC = 4 \frac{1}{2} \]
\[ AC = 6 \]

Notice from the previous example that \( AB + BC = AC \).

Therefore...

**POSTULATE 2**
**SEGMENT ADDITION POSTULATE**

\[ \text{If } B \text{ is between } A \text{ and } C, \text{ then } AB + BC = AC. \]
\[ \text{If } AB + BC = AC, \text{ then } B \text{ is between } A \text{ and } C. \]
Example 3: Use the diagram to find AC.

\[ AC = 13 \]

Example 4: Use the diagram to find DE.

\[ DE = 6 \]

Example 5

Find LM if L is between N and M, \( NL = 6x - 5 \),
\( LM = 2x + 3 \),
and \( NM = 30 \).

\[
(6x - 5) + (2x + 3) = 30
\]

\[
8x - 2 + 2 = 30
\]

\[
8x = 32
\]

\[
x = 4
\]

\[
LM = 11
\]
Example 6
Find MN if N is between M and P,
MN = 3x + 2,
NP = 18,
and MP = 5x.

\[ MN = 3(10) + 2 \]
\[ MN = 32 \]

\[ 3x + 2 + 18 = 5x \]
\[ 3x + 20 = 5x \]
\[ -3x \]
\[ 20 = 2x \]
\[ 2 \]
\[ 10 = x \]

Segments are congruent.
\[ \overline{AB} \cong \overline{CD} \]
"is congruent to"

Segments are equal.
\[ \overline{AB} = \overline{CD} \]
"is equal to"
Are the segments shown in the coordinate plane congruent?

\( RS = 5 \)
\( TU = 4 \)

\( \text{TU and RS have different lengths... so they are not congruent.} \)

**DISTANCE FORMULA**

The distance \( d \) between any two points with coordinates \((x_1, y_1)\) and \((x_2, y_2)\) is given by the formula

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]
Example 7
Find \( PQ \) for \( P(-3, -5) \) and \( Q(4, -6) \).

\[
\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} = \sqrt{(4 - (-3))^2 + (-6 - (-5))^2} = \sqrt{7^2 + (-1)^2} = \sqrt{49 + 1} = \sqrt{50} \approx 5 \cdot \sqrt{2}
\]

Example 8
Find \( JK \) for \( J(-2, -3) \) and \( K(0, -1) \).

\[
\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} = \sqrt{(0 - (-2))^2 + (-1 - (-3))^2} = \sqrt{(2)^2 + 2^2} = \sqrt{4 + 4} = \sqrt{8} \approx 2 \cdot \sqrt{2}
\]
Example 9
Find the length of AC, BC, & DC. Tell whether any of the segments have the same length.

\[
AC = \sqrt{(0+3)^2+(2-8)^2} = \sqrt{9+36} = \sqrt{45} = 3\sqrt{5}
\]

\[
BC = \sqrt{(6-0)^2+(2-5)^2} = \sqrt{36+9} = \sqrt{45} = 3\sqrt{5}
\]

\[
DC = \sqrt{(2-2)^2+(2+4)^2} = \sqrt{0+4} = \sqrt{40} = 2\sqrt{10}
\]

\[
AC \neq BC
\]