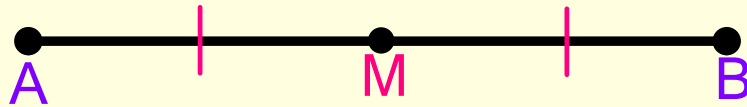


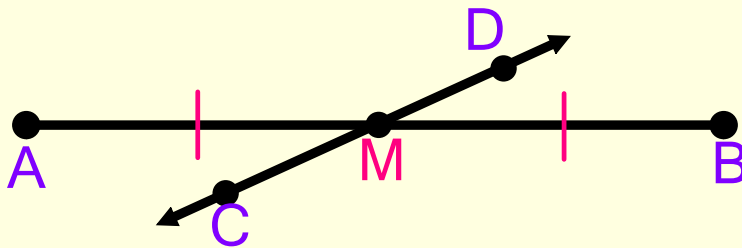
1.3 Use Midpoint & Distance Formulas

The **midpoint** of a segment is the point that divides the segment into two congruent segments.



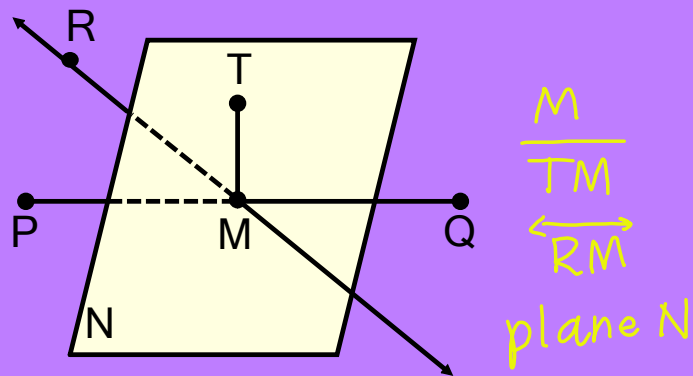
M is the midpoint of \overline{AB} .
So $AM = MB$ and $\overline{AM} = \overline{MB}$.

A **segment bisector** is a point, ray, line, line segment, or plane that intersects the segment at its midpoint.



\overline{CD} is the segment bisector of \overline{AB} .
So $AM = MB$ and $\overline{AM} = \overline{MB}$.

What are the segment bisectors of \overline{PQ} ?



Example 1

If Y is the midpoint of \overline{XZ} ,

$XY = 2a + 11$ and $YZ = 4a - 5$,

find the value of a and the measure of \overline{XZ} .



$$\begin{array}{r} 2a + 11 = 4a - 5 \\ -4a \quad \quad -4a \\ \hline \end{array}$$

$$\begin{array}{r} -2a + 11 = -5 \\ -11 \quad \quad -11 \\ \hline \end{array}$$

$$\begin{array}{r} -2a = -16 \\ -2 \quad \quad -2 \\ \hline \end{array}$$

$$\boxed{a = 8}$$

$$\begin{aligned} XY + YZ &= XZ \\ 2(8) + 11 + 4(8) - 5 &= XZ \\ 16 + 11 + 32 - 5 &= XZ \end{aligned}$$

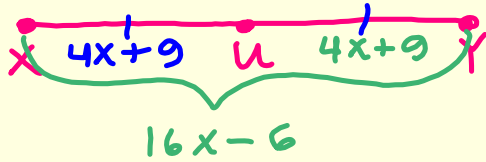
$$\boxed{54 = XZ}$$

Example 2

If U is the midpoint of \overline{XY} ,

$XY = 16x - 6$ and $UY = 4x + 9$,

find the value of x and the measure of \overline{XY} .



$$2(4x+9) = 16x-6$$

$$(4x+9) + (4x+9) = 16x-6$$

$$8x+18 = 16x-6$$

$$\begin{array}{r} -16x \qquad -16x \\ \hline -8x+18 = -6 \\ \qquad -18 \qquad -18 \end{array}$$

$$\begin{array}{r} -8x = -24 \\ \hline -8 \qquad -8 \end{array}$$

$$\boxed{x=3}$$

$$XY = 16(3) - 6$$

$$XY = 48 - 6$$

$$\boxed{XY=42}$$

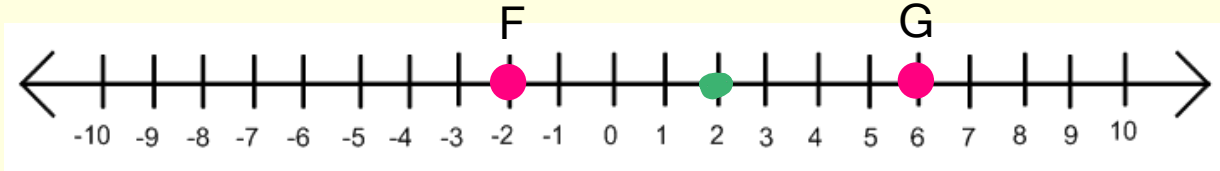
MIDPOINT FORMULAS

1. On a number line, the coordinate of the midpoint of a segment whose endpoints have coordinates a and b is $\frac{a+b}{2}$.
2. In a coordinate plane, the coordinates of the midpoint of a segment whose endpoints have coordinates (x_1, y_1) and (x_2, y_2) are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

Example 3

Use the number line to find the coordinate of the midpoint of \overline{FG} .

$$\frac{-2+6}{2} = \frac{4}{2} = 2$$



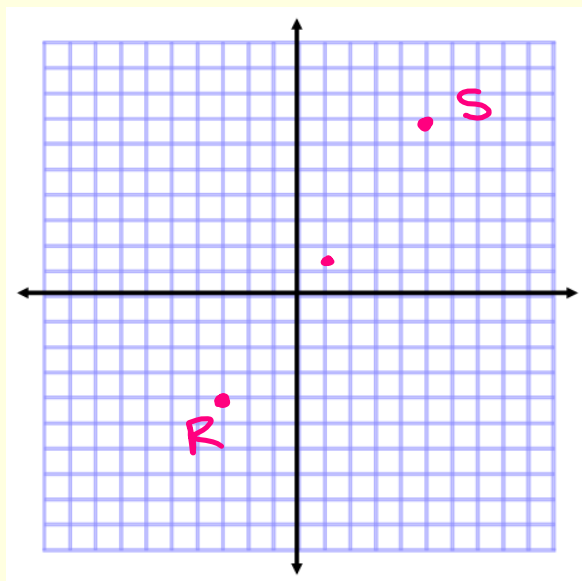
Example 4

If the coordinate of H is -13 and the coordinate of J is 10, what is the coordinate of the midpoint of \overline{HJ} ?

$$\frac{-13+10}{2} = \frac{-3}{2} \text{ or } -1.5$$

Example 5

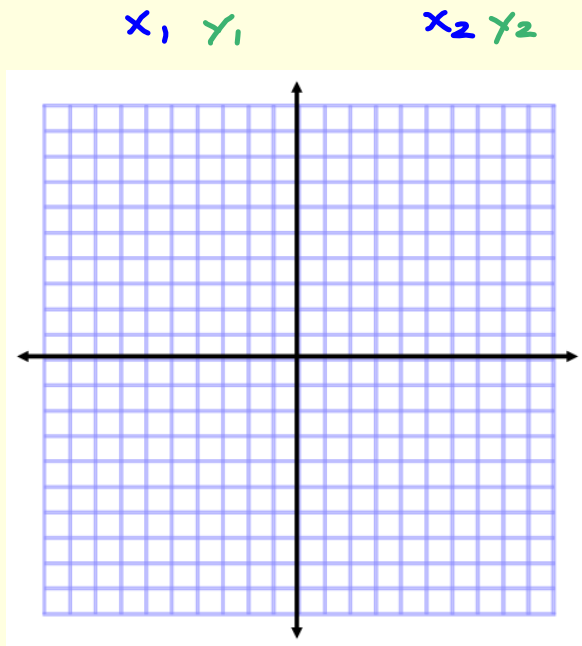
Find the coordinates of Q, the midpoint of \overline{RS} , if the endpoints of \overline{RS} are $R(-3, -4)$ and $S(5, 7)$.



$$\begin{aligned} & \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ & \left(\frac{-3 + 5}{2}, \frac{-4 + 7}{2} \right) \\ & \left(\frac{2}{2}, \frac{3}{2} \right) \\ & (1, 1.5) \end{aligned}$$

Example 6

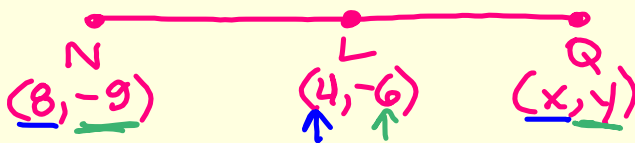
Find the coordinates of the midpoint of \overline{VW} for $V(3, -6)$ and $W(7, 2)$.



$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$
$$\left(\frac{3 + 7}{2}, \frac{-6 + 2}{2} \right)$$
$$\left(\frac{10}{2}, \frac{-4}{2} \right)$$
$$(5, -2)$$

Example 7

Find the coordinates of point Q if L(4, -6) is the midpoint of \overline{NQ} and the coordinates of N are (8, -9).



$$2 \cdot \frac{8 + x}{2} = 4 \cdot 2 \quad 2 \cdot \frac{-9 + y}{2} = -6 \cdot 2$$

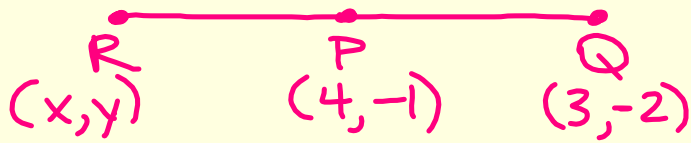
$$\begin{array}{r} 8 + x = 8 \\ -8 \quad -8 \\ \hline x = 0 \end{array}$$

$$\begin{array}{r} -9 + y = -12 \\ +9 \quad +9 \\ \hline y = -3 \end{array}$$

$$Q(0, -3)$$

Example 8

The midpoint of \overline{RQ} is $P(4, -1)$. What are the coordinates of R if Q is at $(3, -2)$?



$$2 \cdot \frac{x+3}{2} = 4 \cdot 2 \quad 2 \cdot \frac{y+(-2)}{2} = -1 \cdot 2$$

$$\begin{array}{r} x+3 = 8 \\ -3 \quad -3 \\ \hline x = 5 \end{array}$$

$$\begin{array}{r} y-2 = -2 \\ +2 \quad +2 \\ \hline y = 0 \end{array}$$

$$R(5, 0)$$