1.3 ALGEBRAIC EXPRESSIONS

variable- a letter that represents a number

constant- a fixed (or specific) number

domain of a variable- set of numbers that the variable is allowed to have

examples: $\sqrt{x}$ has a domain of $x \geq 0$ or $[0, \infty)$ (index is even; cannot be negative)

$\frac{2}{x-3}$ has a domain of $x \neq 3$ or $(-\infty, 3) \cup (3, \infty)$ (cannot be zero)

$x - 3 \neq 0$

$+3 \quad +3$

$x \neq 3$

$\sqrt[3]{x}$ has a domain of all real numbers or $(-\infty, \infty)$ (index is odd)

polynomials- simple algebraic expressions that include addition, subtraction, and multiplication

degree of a polynomial- highest power (exponent) of the variable

degree	degree name
1	linear
2	quadratic
3	cubic
4	quartic
5	quintic

monomial- one term

binomial- two terms

trinomial- three terms

standard form- descending exponential order

Write each sum or difference as a polynomial in standard form. Then classify the polynomial by degree and by number of terms.

1. $(x^3 + x^2 + x + 1) + (2x^3 + 3x^2 + x + 1)$

$3x^3 + 4x^2 + 2x + 2$

cubic polynomial

2. $(1 - 5x + x^3) + (2x^4 - 5x^3 + 10x^2)$

$-2x^4 - 4x^3 + 10x^2 - 5x + 1$

quartic polynomial

Write each sum or difference as a polynomial in standard form. Then classify the polynomial by degree and by number of terms.

3. $(\frac{2}{3}x + \frac{2}{3}x^3 + 1) + (\frac{2}{3} - \frac{1}{3}x^2 - \frac{1}{3}x)$

$\frac{2}{3}x^3 - \frac{1}{3}x^2 + \frac{1}{3}x + \frac{1}{3}$

cubic polynomial

4. $(x^4 + 5x^2 + x) - (x^4 + 2x^3 - x + 4)$

$-2x^3 + 5x^2 + 4$

cubic trinomial

Multiply.

5. $(2x + 1)(3x - 5)$

$6x^2 - 10x + 3x - 5$

$6x^2 - 7x - 5$

6. $(x^2 - 3)(x^3 + 2x + 1)$

$x^2(x^3 + 2x + 1) - 3(x^3 + 2x + 1)$

$x^5 + 2x^3 + x^2 - 3x^3 - 6x - 3$

$x^5 - x^3 + x^2 - 6x - 3$

7. $(1 + \sqrt{x})(2 - 3\sqrt{x})$

$\sqrt{x} \cdot \sqrt{x}$

$(1)(2) + (1)(-3\sqrt{x}) + (\sqrt{x})(2) + (\sqrt{x})(-3\sqrt{x})$

$2 \quad -3\sqrt{x} \quad +2\sqrt{x} \quad -3x$

$2 \quad -\sqrt{x} \quad -3x$

SPECIAL PRODUCTS

diff. of squares

$(a - b)(a + b) = a^2 - b^2$

$(a + b)^2 = a^2 + 2ab + b^2$

$(a - b)^2 = a^2 - 2ab + b^2$

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

Multiply.

8. $(2x - \sqrt{y})(2x + \sqrt{y})$

$(2x)^2 - (\sqrt{y})^2$

$4x^2 - y$

9. $(\overset{a}{3x} + \overset{b}{5})^2$ $a^2 + 2ab + b^2$

$(3x)^2 + 2(3x)(5) + (5)^2$

$9x^2 + 30x + 25$

10. $(\underbrace{2x^2}_{a} - \underbrace{4}_{b})^3$ $a^3 - 3a^2b + 3ab^2 - b^3$

$(2x^2)^3 - 3(2x^2)^2(4) + 3(2x^2)(4)^2 - (4)^3$

$8x^6 - 3(4x^4)(4) + 3(2x^2)(16) - 64$

$8x^6 - 48x^4 + 96x^2 - 64$

FORMULAS/PATTERNS TO KNOW

$x^2 - y^2 = (x-y)(x+y)$

$x^2 + 2xy + y^2 = (x+y)^2$

$x^2 - 2xy + y^2 = (x-y)^2$

$x^3 + y^3 = (x+y)(x^2 - xy + y^2)$ ← do not factor

sum of cubes

$x^3 - y^3 = (x-y)(x^2 + xy + y^2)$ ← do not factor

diff. of cubes

STEPS TO FACTOR POLYNOMIALS

Step 1: Factor out any GCF.

Step 2: For a binomial, check if it is the difference of squares. sum of cubes, diff. of cubes.

Step 3: For a trinomial, check to see if it matches any pattern. If not, jump into a sum/product chart.

Step 4: For 4 terms, use the grouping method.

Step 5: See if any factors can be factored further.

FACTORING QUADRATIC TRINOMIALS

Example: $5x^2 + 17x + 14$ (product, sum, LC)

sum 17 product 70

$\frac{10}{5}$ $\frac{7}{5}$

$\frac{2}{1}$ $\frac{7}{5}$ (LC)

1. Make a sum/product chart.
2. Divide each number by the leading coefficient.
3. Reduce each fraction if possible.
4. Denominator = coefficient for first term (variable)
   Numerator = constant or coefficient of last term

$(x+2)(5x+7)$

FACTOR EACH POLYNOMIAL.

11. $3x^2 - 6x$

$3x(x-2)$

12. $8x^4y^2 + 6x^3y^3 - 2xy^4$

$2xy^2(4x^3 + 3x^2y - y^2)$

FACTOR EACH POLYNOMIAL.

13. $x^2 + 7x + 12$

sum 7 product 12

$4 \quad 3$

$1 \quad 1$

$(x+4)(x+3)$

14. $6x^2 + 7x - 5$

sum 7 product -30

$-3 \quad 10$

$6 \quad 6$

$-1 \quad 5$

$2 \quad 3$

$(2x-1)(3x+5)$

FACTOR EACH POLYNOMIAL.

15. $4x^2 - 25$

$(2x)^2 \quad (5)^2$

$(2x-5)(2x+5)$

16. $27x^3 - 1$

$(3x)^3 \quad (1)^3$

$(x-y)(x^2+xy+y^2)$

$(3x-1)((3x)^2+(3x)(1)+(1)^2)$

$(3x-1)(9x^2+3x+1)$

FACTOR EACH POLYNOMIAL.

17. $x^6 - 8$

18. $x^6 - 16$

FACTOR EACH POLYNOMIAL.

19. $x^2 + 6x + 9$

20. $4x^2 - 4xy + y^2$