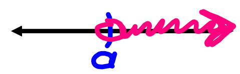
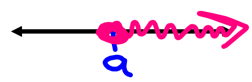
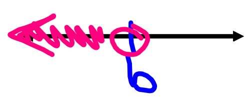
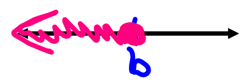
1.1 Real Numbers

A set is a collection of objects, and these objects are called the elements of the set.

Certain sets of real numbers, called intervals, occur frequently in calculus and correspond geometrically to line segments.

Interval Notation	Set description	Graph
(a, b)	$\{x \mid a < x < b\}$	
[a, b]	$\{x \mid a \le x \le b\}$	
[a, b)	$\{x \mid a \le x < b\}$	
(a, b]	$\{x \mid a < x \le b\}$	
(a, ∞)	$\{x \mid x > a\}$	
$[a, \infty)$	$\{x \mid x \ge a\}$	
$(-\infty, b)$	$\{x \mid x < b\}$	
$(-\infty, b]$	$\{x \mid x \le b\}$	
$(-\infty, \infty)$	(set of all real numbers)	

Example: Graph the following intervals on a number line.

a) (-4,5]

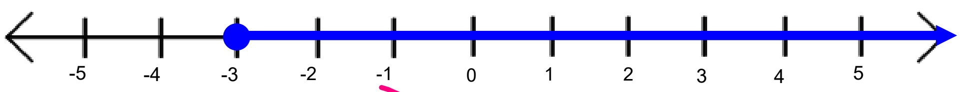
b) [-3,∞)

c) (-∞,2)

d) [-1,3]

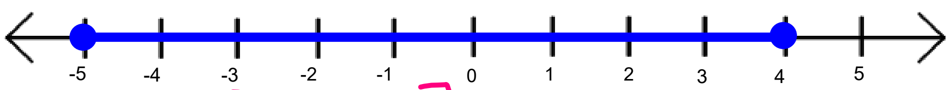
Example: Write the interval for each graph given below.

a)



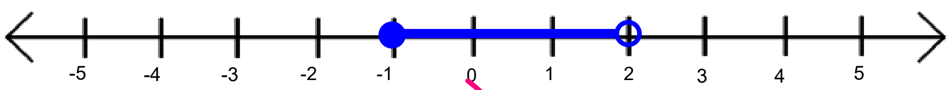
[-3, ∞)

b)



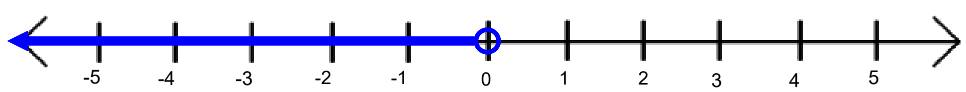
[-5, 4]

c)



[-1, 2)

d)



(-∞, 0)

Absolute Value
- denoted |a|
- the distance from a number to 0 on the real number line.
- Since distance is always positive or 0, |a| ≥ 0 for every number a.

Definition of Absolute Value:

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

Example: Evaluate.

a) $|3| = 3$

b) $|-12| = 12$

c) $|0| = 0$

d) $|\sqrt{2} - 1| = \sqrt{2} - 1$

e) $|1-\sqrt{2}| = \sqrt{2} - 1$

1.2 Exponents and Radicals

KEY CONCEPT *For Your Notebook*

Properties of Exponents

Let a and b be real numbers and let m and n be integers.

Property Name	Definition	Example
Product of Powers	$a^m \cdot a^n = a^{m+n}$	$5^3 \cdot 5^{-1} = 5^{3+(-1)} = 5^2 = 25$
Power of a Power	$(a^m)^n = a^{mn}$	$(3^3)^2 = 3^{3 \cdot 2} = 3^6 = 729$
Power of a Product	$(ab)^m = a^m b^m$	$(2 \cdot 3)^4 = 2^4 \cdot 3^4 = 1296$
Negative Exponent	$a^{-m} = \frac{1}{a^m}, a \neq 0$	$7^{-2} = \frac{1}{7^2} = \frac{1}{49}$
Zero Exponent	$a^0 = 1, a \neq 0$	$(-89)^0 = 1$
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$	$\frac{6^{-3}}{6^{-6}} = 6^{-3-(-6)} = 6^3 = 216$
Power of a Quotient	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$	$\left(\frac{4}{7}\right)^2 = \frac{4^2}{7^2} = \frac{16}{49}$

Example: Evaluate.

a) $x^3x^5 = x^8$

b) $a^{-6}a^2 = a^{-4} = \frac{1}{a^4}$

c) $\frac{c^{10}}{c^{-4}} = c^{14}$

d) $(d^3)^5 = d^{15}$

e) $(2z)^4 = 2^4 z^4 = 16z^4$

f) $(2m^6n)(3mn^4)^3$

$(2m^6n)(27m^3n^{12})$

$54m^9n^{13}$

g) $\frac{6st^{-4}}{2s^{-2}t^2} = 3s^3t^{-6}$

$\frac{3s^3}{t^6}$

Radicals: index $\sqrt[n]{a} = b$ means $b^n = a$

radicand

If n is even, then $a \geq 0$ and $b \geq 0$.

Properties of Radicals:

$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

$\sqrt[2]{\boxed{a \cdot a}} = |a|$

$\sqrt[5]{\boxed{a \cdot a \cdot a \cdot a \cdot a}} = a$

$\sqrt[4]{\boxed{a \cdot a \cdot a \cdot a}} = |a|$

$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

$\sqrt[n]{a^n} = a$ if n is odd

$\sqrt[n]{a^n} = |a|$ if n is even

Example: Simplify.

a) $\sqrt[3]{x^4} = x\sqrt[3]{x}$

$\sqrt[3]{\boxed{x \cdot x \cdot x}\, x}$

b) $\sqrt[4]{81x^8y^4} = 3x^2|y|$

$\sqrt[4]{81\boxed{x \cdot x \cdot x \cdot x}\boxed{x \cdot x \cdot x \cdot x}\boxed{y \cdot y \cdot y \cdot y}}$

Adding and Subtracting Radicals:

Only LIKE radicals can be combined!

(LIKE radicals have the same index and radicand.)

Example: Simplify.

a) $\sqrt{32} + \sqrt{200}$

$\sqrt{\boxed{2 \cdot 2}\boxed{2 \cdot 2}\, 2} \qquad \sqrt{2 \cdot 100}$

$4\sqrt{2} + 10\sqrt{2}$

$14\sqrt{2}$

b) $\sqrt{25b} - \sqrt{b^3}$

$\sqrt{\boxed{b \cdot b} \cdot b}$

$5\sqrt{b} - b\sqrt{b}$

$\sqrt{b}(5 - b)$

Rational (Fractional) Exponents are equivalent to radicals.

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

index

outside exp.

$$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$$

Example: Simplify.

a) $4^{1/2}$

$\left(\sqrt{4}\right)^1$

2

b) $8^{2/3}$

$\left(\sqrt[3]{8}\right)^2$

$(2)^2$

4

c) $125^{-1/3}$

$\frac{1}{125^{1/3}}$

$\frac{1}{\left(\sqrt[3]{125}\right)^1}$

$\frac{1}{5}$

Example: Simplify. Write in radical notation.

a) $a^{1/3} \cdot a^{7/3}$

$a^{\frac{8}{3}}$

$(\sqrt[3]{a})^8$

b) $\dfrac{k^{2/5}k^{7/5}}{k^{3/5}} = \dfrac{k^{\frac{9}{5}}}{k^{\frac{3}{5}}}$

$k^{\frac{6}{5}}$

$(\sqrt[5]{k})^6$

c) $(2f^3g^4)^{3/2}$

$2^{\frac{3}{2}}(f^3)^{\frac{3}{2}}(g^4)^{\frac{3}{2}}$

$\underbrace{2^{\frac{3}{2}}f^{\frac{9}{2}}}g^6$

$\sqrt{2^3f^9} \cdot g^6$

$2f^4g^6\sqrt{2f}$

Rationalizing the Denominator

Eliminating the radical in the denominator by multiplying both numerator and denominator by an appropriate expression

Example: Simplify.

a) $\dfrac{2}{\sqrt[2]{3}} \cdot \dfrac{\sqrt{3}}{\boxed{\sqrt{3}}}$

$\dfrac{2\sqrt{3}}{3}$

b) $\dfrac{1}{\sqrt[3]{x^2}} \cdot \dfrac{\sqrt[3]{x}}{\boxed{\sqrt[3]{x}}}$

$\dfrac{\sqrt[3]{x}}{x}$

c) $\sqrt[7]{\dfrac{1}{a^2}}$

$\dfrac{\sqrt[7]{1}}{\sqrt[7]{a^2}} \cdot \dfrac{\sqrt[7]{a^5}}{\sqrt[7]{a^5}}$

$\dfrac{\sqrt[7]{a^5}}{a}$