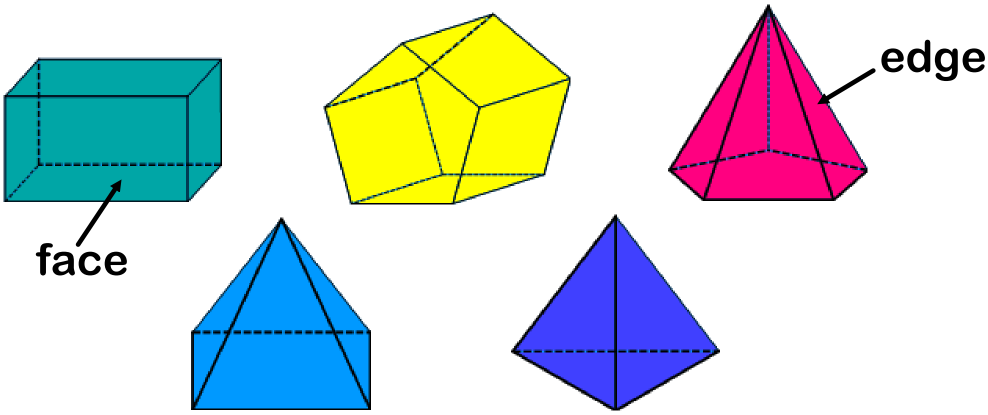
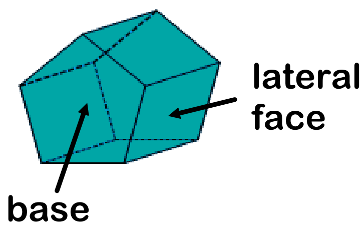
EXPLORING 3-DIMENSIONAL FIGURES

All of the surfaces of the solids so far are flat surfaces called faces. Solids with all flat surfaces that enclose a single region of space are called polyhedrons or polyhedra. All of the faces are polygons, and the line segments where the faces intersect are called edges.



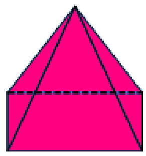
A prism is a polyhedron with two congruent faces that are polygons in parallel planes, that are called bases. The other faces, called lateral faces, are shaped like parallelograms.



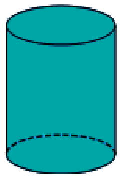
Prisms are named by the shape of their bases. The one above is a pentagonal prism.

A regular prism is a prism whose bases are regular polygons, such as in a cube.

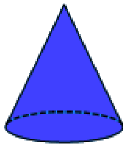
A polyhedron that has all faces except one intersecting at one point is a pyramid.



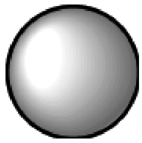
A cylinder is like a prism, but the bases are circular. This means that it is not a polyhedron.



A cone is like a pyramid, but the base is circular. It is also not a polyhedron.

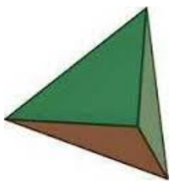


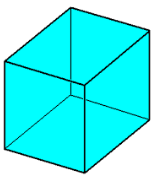
A sphere is a set of points in space that are a given distance from a given point.



A polyhedron is regular if all of its faces are shaped like congruent regular polygons. There are exactly five types of regular polyhedra, and they are called Platonic solids because Plato described them so fully in his writings.

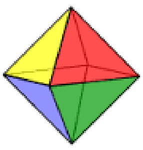
4 faces
tetrahedron



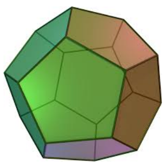


6 faces
hexahedron

8 faces
octahedron

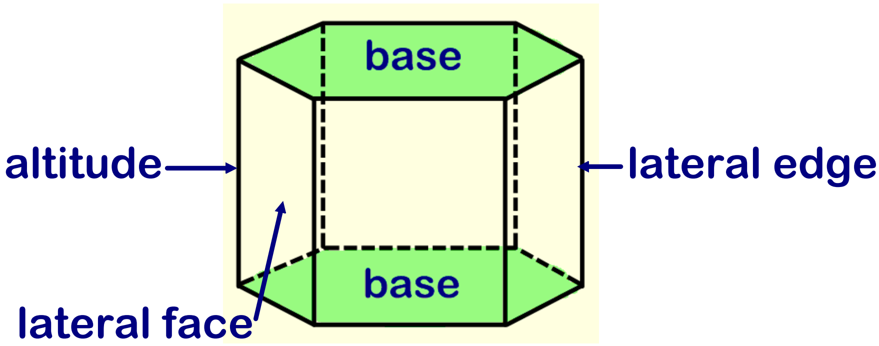


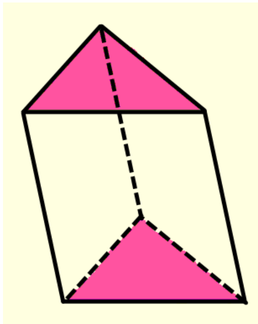
12 faces
dodecahedron



20 faces
icosahedron

SURFACE AREA OF PRISMS & CYLINDERS





A segment perpendicular to the planes containing the two bases, with an endpoint in each plane, is called the altitude. The length of the altitude is called the height. A prism whose lateral edges are also altitudes is called a right prism. If a prism is not right, then it is an oblique prism.

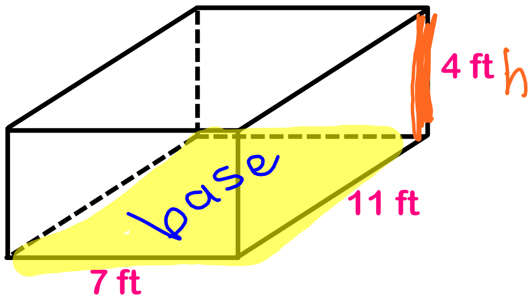
surface area: the area of all the faces of a prism (includes bases)

Surface Area of a Right Prism

$T = Ph + 2B$ where T = total surface area
P = perimeter of base
h = height of prism
B = area of base

Example 1

Find the surface area of the rectangular prism.



$T = Ph + 2B$

$T = (36)(4) + 2(77)$

$\boxed{T = 298\ ft^2}$

$P = 7 + 11 + 7 + 11$

$P = 36$

$h = 4$

$B = lw$ rectangle

$B = 7 \cdot 11$

$B = 77$

Example 2

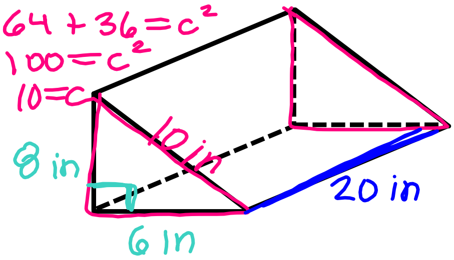
Find the surface area of a right triangular prism with a height of 20 inches and a right triangular base with legs of 8 and 6 inches.

$8^2 + 6^2 = c^2$

$64 + 36 = c^2$

$100 = c^2$

$10 = c$



$T = Ph + 2B$

$T = (24)(20) + 2(24)$

$\boxed{T = 528\ in^2}$

$P = 8 + 6 + 10 = 24$

$h = 20$

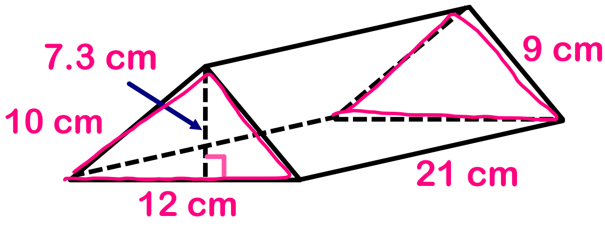
$B = \frac{1}{2} \cdot b \cdot h$ triangle

$B = \frac{1}{2} \cdot 8 \cdot 6$

$B = 24$

Example 3

Find the surface area of the triangular prism.



$T = Ph + 2B$

$T = (31)(21) + 2(43.8)$

$\boxed{T = 738.6\ cm^2}$

$P = 10 + 12 + 9 = 31$

$h = 21$

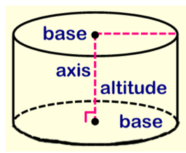
triangle

$B = \frac{1}{2} \cdot b \cdot h$

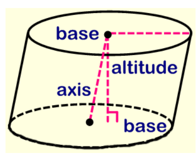
$B = \frac{1}{2} \cdot 12 \cdot 7.3$

$B = 43.8$

A **cylinder** is like a prism but with circular bases. The **axis** of the cylinder is the segment whose endpoints are centers of the bases. The **altitude** is a segment perpendicular to the planes containing the bases with an endpoint in each plane.



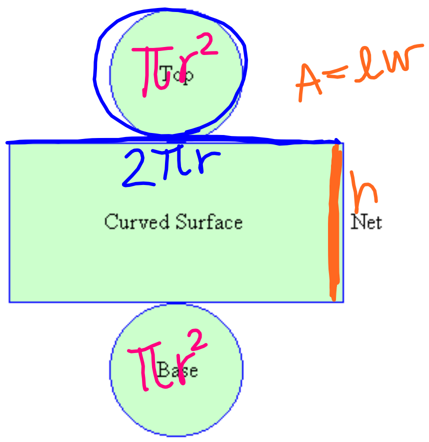
right cylinder



oblique cylinder

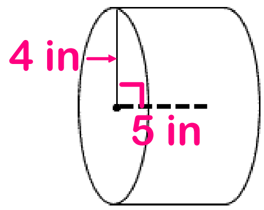
Surface Area of a Cylinder

$T = 2\pi rh + 2\pi r^2$



Example 4

Find the surface area of the right cylinder.



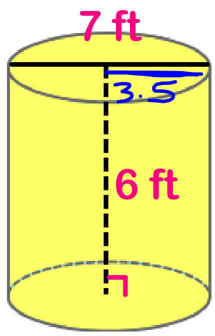
$T = 2\pi rh + 2\pi r^2$

$T = 2\pi(4)(5) + 2\pi(4)^2$

$\boxed{T \approx 226.19 \text{ in}^2}$

Example 5

Find the surface area of the right cylinder.



$T = 2\pi rh + 2\pi r^2$

$T = 2\pi(3.5)(6) + 2\pi(3.5)^2$

$\boxed{T \approx 208.92 \text{ ft}^2}$

Example 6

The surface area of a right cylinder is 200 square centimeters. If the diameter of the base is 10 centimeters, find the height of the cylinder.

T

$r = 5$

$h = ?$

$$T = 2\pi r h + 2\pi r^2$$

$$200 = 2\pi(5)h + 2\pi\overset{25}{(5)^2}$$

$$200 = 10\pi \boxed{h} + 50\pi$$

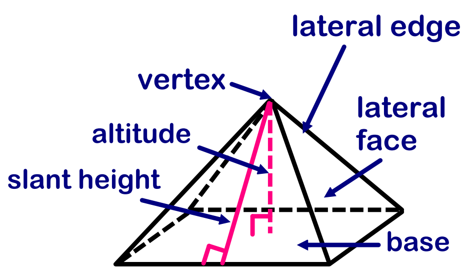
$$-50\pi \qquad -50\pi$$

$$\frac{200 - 50\pi}{10\pi} = \frac{10\pi h}{10\pi}$$

$$(200 - 50\pi) \div (10\pi)$$

$$\boxed{1.37\text{cm} \approx h}$$

SURFACE AREA OF PYRAMIDS



REGULAR PYRAMID

vertex- all faces except one intersect at this point

base- face that does not intersect the other faces at the vertex (always a polygon)

lateral faces- faces that intersect at the vertex and form triangles

altitude- the segment from the vertex perpendicular to the base

slant height- the height of each lateral face

Surface Area of a Regular Pyramid

$T = \frac{1}{2}Pl + B$

where P is the perimeter of the base
l is the slant height
B is the area of the base

Example 7

Find the surface of the pyramid below with a square base.

$T = \frac{1}{2}P\ell + B$

$T = \frac{1}{2}(40)(12) + 100$

$\boxed{T = 340\ cm^2}$

$P = 4(10) = 40$

$B = s^2$

$B = 10^2$

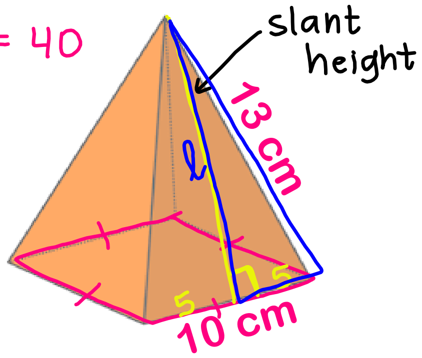
$B = 100$

$5^2 + \ell^2 = 13^2$

$25 + \ell^2 = 169$

$\ell^2 = 144$

$\ell = 12$



Example 8

Find the surface area of the pyramid with a square base as shown below.

$T = \frac{1}{2}P\ell + B$

$T = \frac{1}{2}(64)(17) + 256$

$\boxed{T = 800 m^2}$

$x^2 + 15^2 = 17^2$

$x^2 + 225 = 289$

$x^2 = 64$

$x = 8$

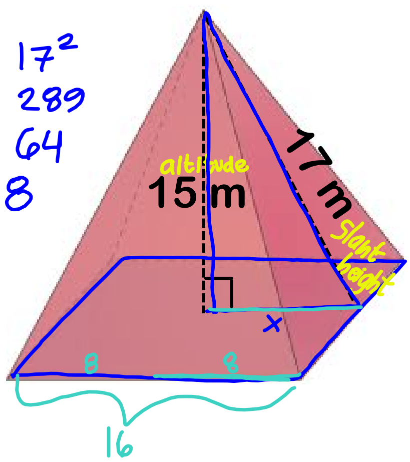
$P = 4(16) = 64$

$\ell = 17$

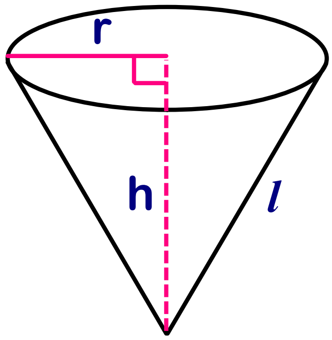
$B = s^2$

$B = 16^2$

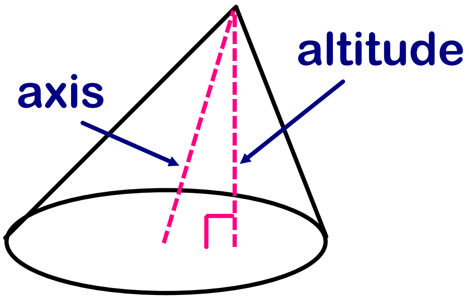
$B = 256$



SURFACE AREA OF CONES



RIGHT CONE



OBLIQUE CONE

Surface Area of a Right Circular Cone

$T = \pi r l + \pi r^2$

where r is the radius of the base

l is the slant height

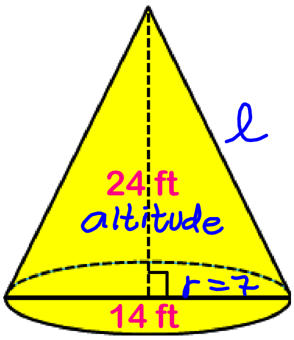
Example 9

Find the surface area of the cone.

$T = \pi r \ell + \pi r^2$

$T = \pi(7)(25) + \pi(7)^2$

$\boxed{T \approx 703.72 \text{ ft}^2}$



$7^2 + 24^2 = \ell^2$

$49 + 576 = \ell^2$

$625 = \ell^2$

$25 = \ell$

SURFACE AREA OF A SPHERE

$T = 4\pi r^2$

Example 10

Find the surface area of an NCAA basketball that has a radius of $4\frac{3}{4}$ inches.

$T = 4\pi r^2$

$T = 4\pi\left(\frac{19}{4}\right)^2$

$\boxed{T \approx 283.53 \text{ in}^2}$

Example 11

Find the surface area of an Olympic-sized volleyball that has a circumference of 27 inches.

$T = 4\pi r^2$

$T = 4\pi\left(\frac{27}{2\pi}\right)^2$

$\boxed{T \approx 232.05 \text{ in}^2}$

$C = 2\pi r$

$\frac{27}{2\pi} = \frac{2\pi r}{2\pi}$

$\frac{27}{2\pi} = r$