# 7.1 Part 2 Trigonometric Identities

Sometimes, we transform each side
of the equation separately,
to arrive at the same result.

Warning! Warning! Warning! Warning!
To prove an identity, we do NOT just
perform the same operations
on both sides of the equation.

Example 1
Verify the identity.

$$\frac{1 + \cos x}{\cos x} = \frac{\tan^2 x}{\sec x - 1}$$

$$= \frac{\sec^2 x - 1}{\sec x - 1}$$

$$= \frac{\cancel{(\sec x - 1)}(\sec x + 1)}{\cancel{\sec x - 1}}$$

$$= \sec x + 1$$

$$= \frac{1}{\cos x} + \frac{\cos x}{\cos x}$$

$$\checkmark \frac{1 + \cos x}{\cos x} = \frac{1 + \cos x}{\cos x}$$

## Example 2

Verify the identity.

$$\frac{\cos u}{\sec u \sin u} = \csc u - \sin u$$

$$\frac{\cos u}{\frac{1}{\cos u}\sin u} =$$

$$\frac{\cos u}{\frac{\sin u}{\cos u}} =$$

$$\cos u \cdot \frac{\cos u}{\sin u} =$$

$$\frac{\cos^2 u}{\sin u} =$$

$$\frac{1-\sin^2 u}{\sin u} =$$

$$\frac{1}{\sin u} - \frac{\sin^2 u}{\sin u} =$$

$$\csc u - \sin u = \csc u - \sin u \checkmark$$

## Example 3

Verify the identity.

$$\underset{\text{even}}{\cos(-x)} - \underset{\text{odd}}{\sin(-x)} = \cos x + \sin x$$

$$\cos x - -\sin x =$$

$$\cos x + \sin x = \cos x + \sin x \checkmark$$

Example 4

Verify the identity.

$\csc x[\csc x + \sin(-x)] = \cot^2 x$

$\csc x[\csc x - \sin x] =$

$\csc^2 x - \csc x \sin x =$

$\csc^2 x - 1 =$

$1 + \cot^2 x - 1 =$

$\cot^2 x = \cot^2 x$ ✓

Example 5

Verify the identity.

$(\sin x + \cos x)^2 = 1 + 2 \sin x \cos x$

$(\sin x + \cos x)(\sin x + \cos x) =$

$\sin^2 x + \sin x \cos x + \sin x \cos x + \cos^2 x =$

$1 + 2 \sin x \cos x = 1 + 2 \sin x \cos x$ ✓

## Example 6

Verify the identity.

$$\frac{\cos x}{\sec x} + \frac{\sin x}{\csc x} = 1$$

$$\frac{\cos x}{\frac{1}{\cos x}} + \frac{\sin x}{\frac{1}{\sin x}} =$$

$$\cos x \cdot \frac{\cos x}{1} + \sin x \cdot \frac{\sin x}{1} =$$

$$\cos^2 x + \sin^2 x =$$

$$1 = 1 \checkmark$$

## Example 7

Verify the identity.

$$(\sin x + \cos x)^4 = (1 + 2 \sin x \cos x)^2$$

$$\left[(\sin x + \cos x)^2\right]^2 =$$

↓ ex 5

$$(1 + 2\sin x \cos x)^2 = (1 + 2\sin x \cos x)^2 \checkmark$$

Example 8

Verify the identity.

$$\frac{1 - \sin x}{1 + \sin x} = (\sec x - \tan x)^2$$

$$= (\sec x - \tan x)(\sec x - \tan x)$$

$$= \sec^2 x - \tan x \sec x - \tan x \sec x + \tan^2 x$$

$$= \sec^2 x - 2\tan x \sec x + \tan^2 x$$

$$= \frac{1}{\cos^2 x} - 2 \cdot \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} + \frac{\sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x} - \frac{2\sin x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x}$$

$$= \frac{1 - 2\sin x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1 - 2\sin x + \sin^2 x}{1 - \sin^2 x}$$

$$= \frac{(1 - \sin x)\cancel{(1 - \sin x)}}{(1 + \sin x)\cancel{(1 - \sin x)}}$$

$$\checkmark \quad \frac{1 - \sin x}{1 + \sin x} = \frac{1 - \sin x}{1 + \sin x}$$

Example 9

Verify the identity.

$$\frac{\sin A}{1 - \cos A} - \cot A = \csc A$$

$$\frac{\sin^2 A}{\sin A\,(1 - \cos A)} - \frac{\cos A\,(1 - \cos A)}{\sin A\,(1 - \cos A)} =$$

$$\frac{\sin^2 A}{\sin A(1 - \cos A)} + \frac{-\cos A + \cos^2 A}{\sin A(1 - \cos A)} =$$

$$\frac{\sin^2 A - \cos A + \cos^2 A}{\sin A(1 - \cos A)} =$$

$$\frac{\cancel{1 - \cos A}}{\sin A\cancel{(1 - \cos A)}} =$$

$$\frac{1}{\sin A} =$$

$$\csc A = \csc A \checkmark$$