7.1 Part 1 Trigonometric Identities

Review of Basic Trigonometric Identities

Reciprocal Identities

$$\csc x = \frac{1}{\sin x} \quad \sec x = \frac{1}{\cos x} \quad \cot x = \frac{1}{\tan x}$$

$$\tan x = \frac{\sin x}{\cos x} \quad \cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$\tan^2 x + 1 = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

Even-Odd Identities

$$\sin(-x) = -\sin x$$

$$\cos(-x) = \cos x$$

$$\tan(-x) = -\tan x$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - u\right) = \cos u$$

$$\cos\left(\frac{\pi}{2} - u\right) = \sin u$$

$$\tan\left(\frac{\pi}{2} - u\right) = \cot u$$

$$\cot\left(\frac{\pi}{2} - u\right) = \tan u$$

$$\sec\left(\frac{\pi}{2} - u\right) = \csc u$$

$$\csc\left(\frac{\pi}{2} - u\right) = \sec u$$

Identities enable us to write the same expression in different ways. It is often possible to write a complicated looking expression as a much simpler one. To simplify trigonometric expressions, we use factoring, common denominators, the Special Product formulas, and the fundamental trigonometric identities.

Example 1

Simplify the expression $\cos t + \tan t \sin t$.

$$\cos t + \frac{\sin t}{\cos t} \cdot \sin t$$

$$\frac{\cos t}{\cos t} \cdot \cos t + \frac{\sin^2 t}{\cos t}$$

$$\frac{\cos^2 t}{\cos t} + \frac{\sin^2 t}{\cos t}$$

$$\frac{\cos^2 t + \sin^2 t}{\cos t}$$

$$\frac{1}{\cos t}$$

$$\sec t$$

Example 2

Simplify by combining fractions.

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta \cdot \cos\theta}{(1+\sin\theta)\cos\theta}$$

$$\frac{\sin\theta + \sin^2\theta}{\cos\theta(1+\sin\theta)} + \frac{\cos^2\theta}{\cos\theta(1+\sin\theta)}$$

$$\frac{\sin\theta + \sin^2\theta + \cos^2\theta}{\cos\theta(1+\sin\theta)} \qquad \sin^2\theta + \cos^2\theta = 1$$

$$\frac{\sin\theta + 1}{\cos\theta(1+\sin\theta)}$$

$$\frac{1}{\cos\theta}$$

$$\sec\theta$$

Example 3

Write the trigonometric expression in terms of sine and cosine, and then simplify.

a) $\sin x \sec x$

$\sin x \cdot \frac{1}{\cos x}$

$\frac{\sin x}{\cos x}$

$\tan x$

b) $\sin x \cos x \sec x$

$\sin x \cdot \cos x \cdot \frac{1}{\cos x}$

$\sin x$

Example 4

Write the trigonometric expression in terms of sine and cosine, and then simplify.

a) $\frac{\sec x}{\csc x}$ $\frac{\frac{1}{\cos x}}{\frac{1}{\sin x}} = \frac{1}{\cos x} \cdot \frac{\sin x}{1} = \frac{\sin x}{\cos x} = \tan x$

b) $\cos^2 x(1 + \tan^2 x)$

$\cos^2 x \left(1 + \frac{\sin^2 x}{\cos^2 x}\right)$

$\cos^2 x + \sin^2 x$

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Tips for Proving Trigonometric Identities

1. Start with one side. It's usually easier to start with the more complicated side.

2. Use known identities. Use algebra and the identities that you know to change the side you started with. Bring fractional expressions to a common denominator, factor, and use the fundamental identities to simplify expressions.

3. Convert to sines and cosines. If you are stuck, you find it helpful to rewrite all functions in terms of sines and cosines.

Example 5

Verify the identity.

$\cos\theta(\sec\theta - \cos\theta) = \sin^2\theta$

$\cos\theta \sec\theta - \cos^2\theta =$

$1 - \cos^2\theta =$

$\sin^2\theta = \sin^2\theta$ ✓

Example 6

Verify the identity.

$$2\tan x \sec x = \frac{1}{1 - \sin x} - \frac{1}{1 + \sin x}$$

$$= \frac{1+\sin x}{(1-\sin x)(1+\sin x)} + \frac{-1+\sin x}{(1-\sin x)(1+\sin x)}$$

$$= \frac{2\sin x}{(1-\sin x)(1+\sin x)} \leftarrow \text{FOIL}$$

$$= \frac{2\sin x}{1-\sin^2 x}$$

$$= \frac{2\sin x}{\cos^2 x}$$

$$= \frac{2}{\cos x}\cdot\frac{\sin x}{\cos x}$$

$$2\tan x \sec x = 2\tan x \sec x \checkmark$$

Example 7

Verify the identity.

$$\frac{\cos u}{1 - \sin u} = \sec u + \tan u$$

mult. by conjugate

$$\frac{\cos u}{1-\sin u}\cdot\frac{1+\sin u}{1+\sin u} =$$

$$\frac{\cos u + \cos u \sin u}{1-\sin^2 u} =$$

$$\frac{\cos u + \cos u \sin u}{\cos^2 u} =$$

$$\frac{\cos u}{\cos^2 u} + \frac{\cos u \sin u}{\cos^2 u} =$$

$$\frac{1}{\cos u} + \frac{\sin u}{\cos u}$$

$$\sec u + \tan u = \sec u + \tan u \checkmark$$