9.2 Part 2 Parabolas

axis of symmetry: x-axis

STANDARD EQUATION OF A PARABOLA WITH VERTEX (0,0)

$$x = \frac{1}{4p}y^2$$

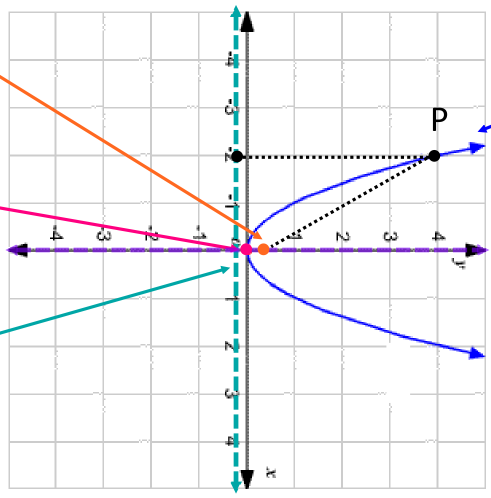
p > 0: opens right

p < 0: opens left

focus: (p,0) lies on the axis of symmetry

vertex: (0,0)

directrix: x = -p perpendicular to the axis of symmetry



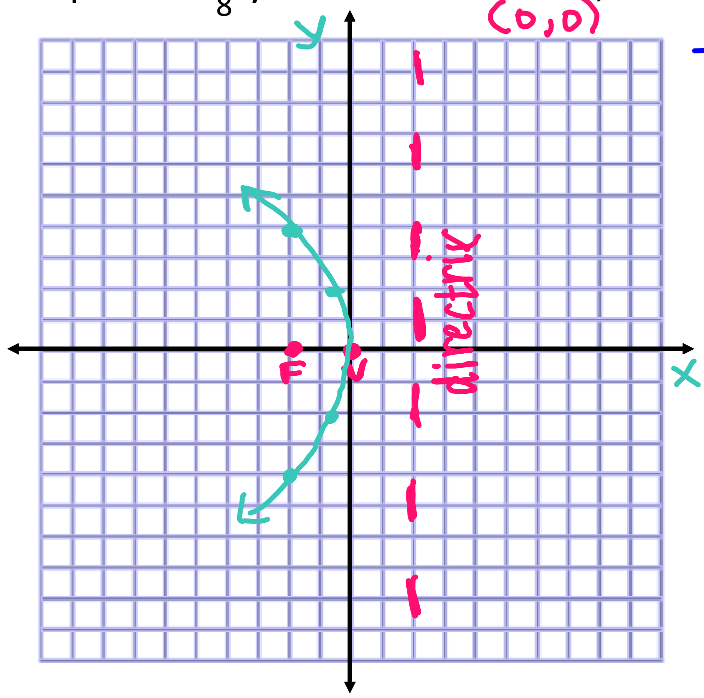
The distance between the **vertex and focus** and **vertex and directrix** is the same. This distance is called p.

Example 1

Graph $x = -\frac{1}{8}y^2$. Label the vertex, focus, and directrix.

x=2

(0,0)



$$-\frac{1}{8} = \frac{1}{4p}$$

$$4p = -8$$

$$p = -2 \quad \text{opens left}$$

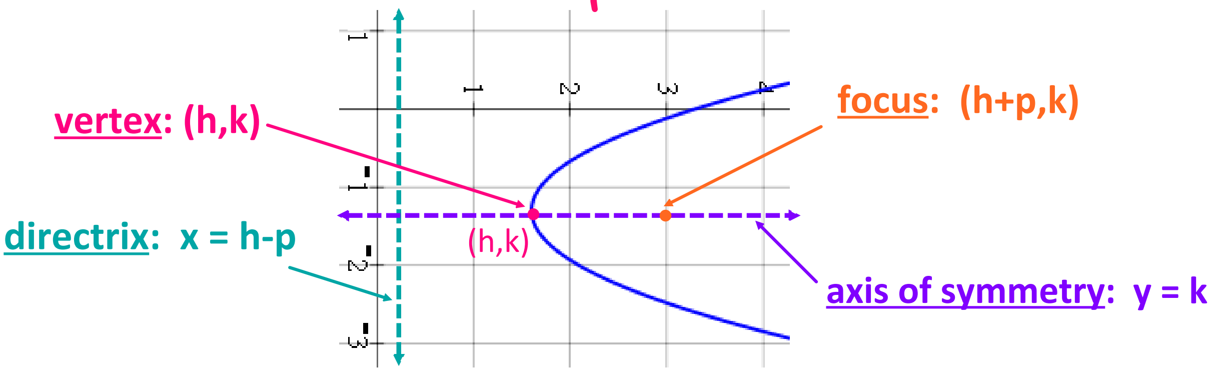
x	y
-2	4
$-\frac{1}{2}$	2
0	0
$-\frac{1}{2}$	-2
-2	-4

Standard Equation of a Translated Parabola

(meaning its vertex is not at the origin)

$$x - h = \frac{1}{4p}(y - k)^2$$

$$x = \frac{1}{4p}(y - k)^2 + h$$



Example 2

$x = \frac{1}{12}(y+2)^2 + 1$

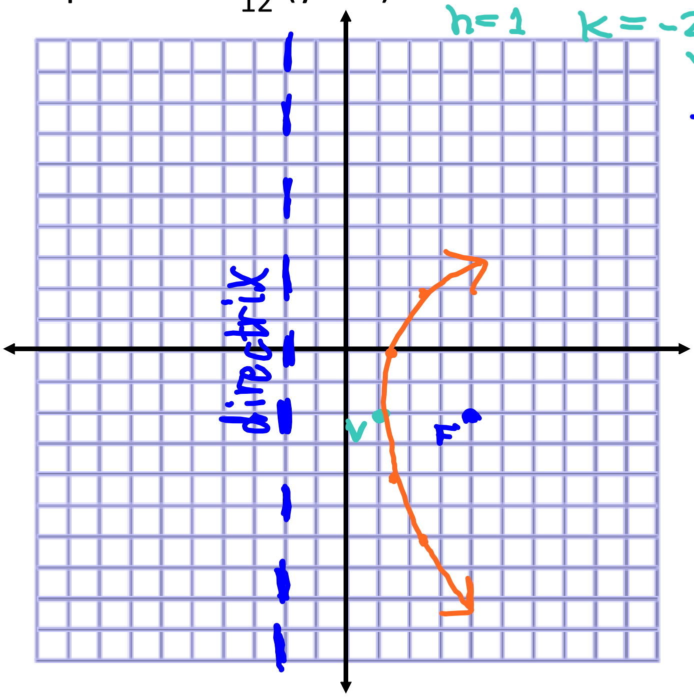
h → exactly what you see
k → opposite

Graph $x - 1 = \frac{1}{12}(y + 2)^2$. Label the vertex, focus, and directrix.

$h = 1$ $k = -2$

vertex $(1, -2)$

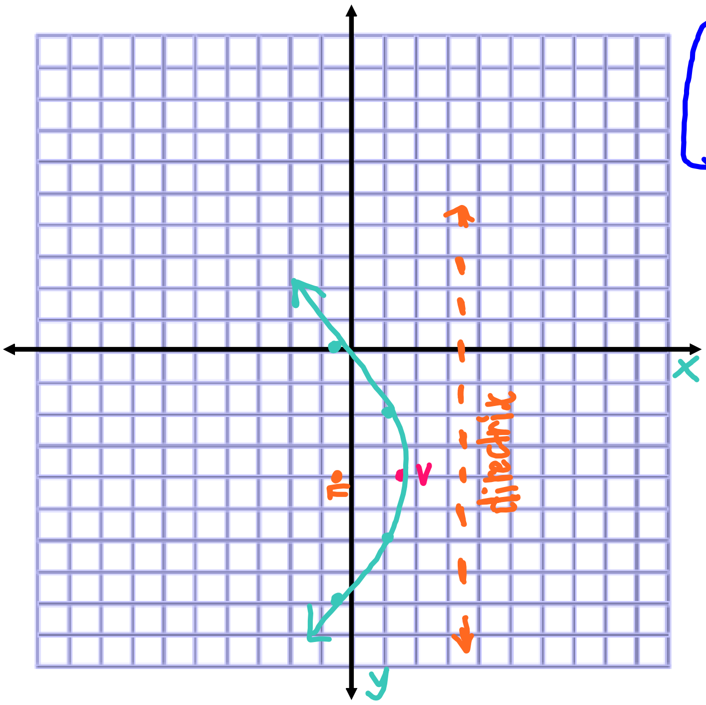
$\frac{1}{12} = \frac{1}{4p}$ $p = 3$ → opens right



x	y
$2\frac{1}{3}$	-6
$1\frac{1}{3}$	-4
1	-2
$1\frac{1}{3}$	0
$2\frac{1}{3}$	2

Example 3

Rewrite the equation $y^2 - 8y + 8x + 4 = 0$ in standard form. Then graph. Label the vertex, focus, and directrix.



$y^2 + 8y \pm 16 = -8x - 4 \pm 16$

$\frac{1}{2}(8) = 4$

$(4)^2 = 16$

$(y+4)^2 = -8x + 12$

$-12 \qquad -12$

$\frac{(y+4)^2}{-8} - \frac{12}{-8} = \frac{-8x}{-8}$

$-\frac{1}{8}(y+4)^2 + \frac{3}{2} = x$

h, k

vertex $\left(\frac{3}{2}, -4\right)$

$\frac{-1}{8} = \frac{1}{4p}$

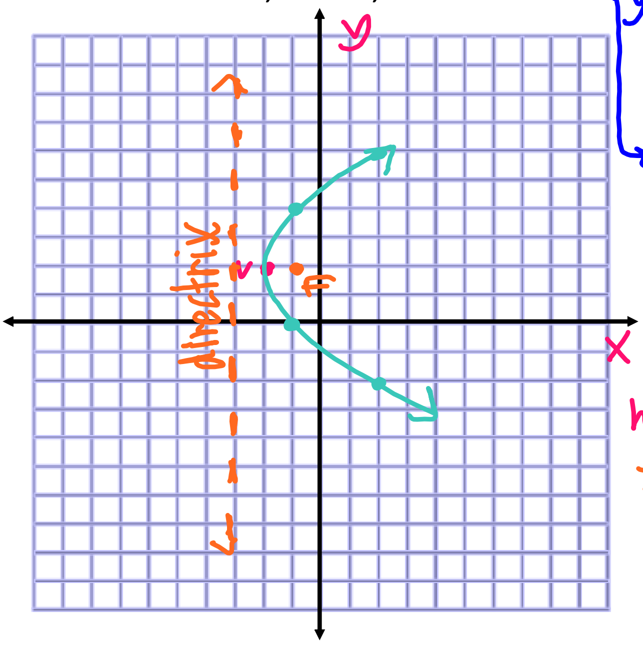
$p = -2$

Left

x	y
$-\frac{1}{2}$	-8
1	-6
$\frac{3}{2}$	-4
1	-2
$-\frac{1}{2}$	0

Example 4

Rewrite the equation $y^2 - 4y - 4x = 4$ in standard form. Then graph. Label the vertex, focus, and directrix.



$y^2 - 4y \pm 4 = 4x + 4 \pm 4$

$\frac{1}{2}(-4) = -2$

$(-2)^2 = 4$

$(y-2)^2 = 4x + 8$

$-8 \qquad -8$

$\frac{(y-2)^2}{4} - \frac{8}{4} = \frac{4x}{4}$

$\frac{1}{4}(y-2)^2 - 2 = x$

$h = -2 \quad k = 2 \quad$ vertex $(-2, 2)$

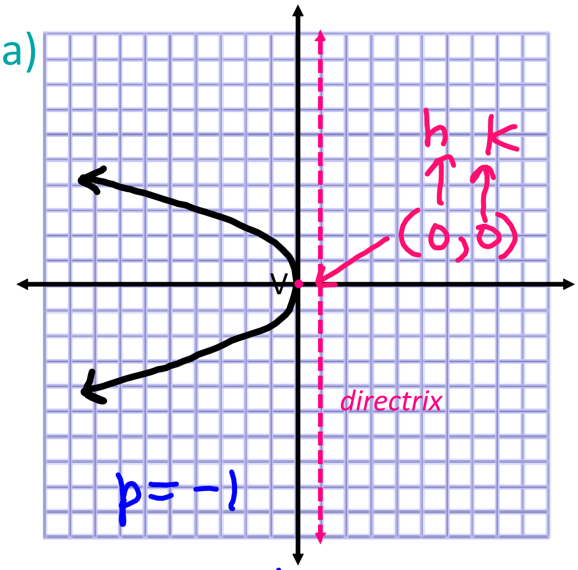
$\frac{1}{4} = \frac{1}{4p} \quad p = 1 \quad$ right

x	y
2	6
-1	4
-2	2
-1	0
2	-2

Example 5

Write the standard equation for each parabola graphed.

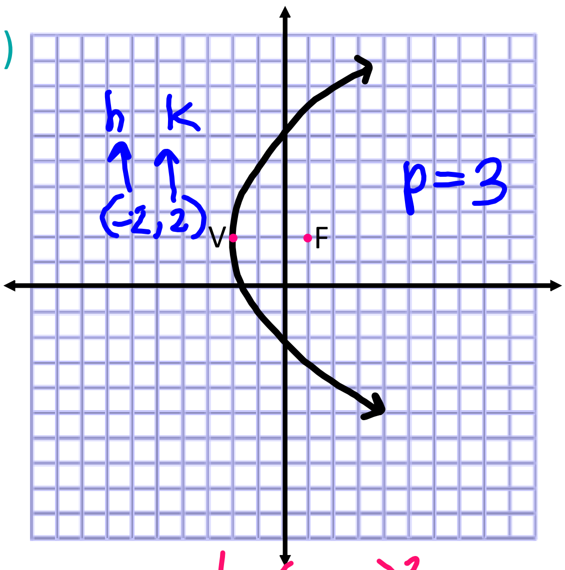
a)



$$x = \frac{1}{4(-1)} y^2$$

$$\boxed{x = -\frac{1}{4} y^2}$$

b)



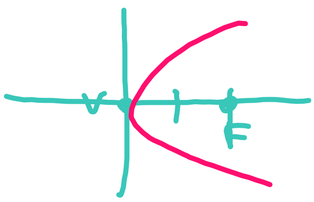
$$x = \frac{1}{4(3)} (y-2)^2 - 2$$

$$x = \frac{1}{12} (y-2)^2 - 2$$

Example 6

Write the standard equation for each parabola with the given characteristics.

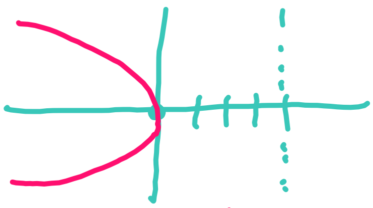
a) ~~origin~~ vertex: (0,0)
focus: (2,0)



$p = 2$

$$\boxed{x = \frac{1}{8} y^2}$$

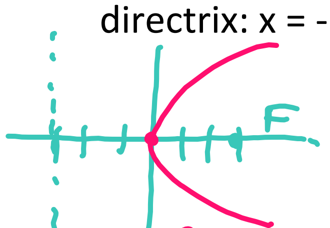
b) ~~origin~~ vertex: (0,0)
directrix: x = 4



$p = -4$

$$\boxed{x = -\frac{1}{16} y^2}$$

c) focus: (3,0)
directrix: x = -3



$p = 3$

$$\boxed{x = \frac{1}{12} y^2}$$