

8.4 Part 2 Zero Product Property & Vertical Motion Model

Zero Product Property

For all numbers a and b , if $ab = 0$, then $a = 0$, $b = 0$, or both a and b equal 0.

Set
each
factor
= 0

Example 1: Solve $(y + 2)(3y + 5) = 0$. Then check each solution.

$$\begin{array}{r} y + 2 = 0 \\ \underline{-2 \quad -2} \\ \boxed{y = -2} \end{array} \qquad \begin{array}{r} 3y + 5 = 0 \\ \underline{-5 \quad -5} \\ 3y = -5 \\ \underline{\quad \quad 3} \\ \boxed{y = -\frac{5}{3}} \end{array}$$

Example 2: Solve $(2a + 4)(a - 9) = 0$. Then check each solution.

$$\begin{array}{r} 2a + 4 = 0 \\ \underline{-4 \quad -4} \\ 2a = -4 \\ \underline{\quad \quad 2} \\ \boxed{a = -2} \end{array} \qquad \begin{array}{r} a - 9 = 0 \\ \underline{+9 \quad +9} \\ \boxed{a = 9} \end{array}$$

↓

$$\begin{array}{r} (2 \cdot -2 + 4)(-2 - 9) \stackrel{?}{=} 0 \\ \downarrow \quad \downarrow \\ 0 \cdot -11 = 0 \checkmark \end{array}$$

↓

$$\begin{array}{r} (2 \cdot 9 + 4)(9 - 9) \stackrel{?}{=} 0 \\ \downarrow \quad \downarrow \\ 22 \cdot 0 = 0 \checkmark \end{array}$$

Example 3: Solve $k^2 - 11k = 0$.

$$\begin{array}{r} k \cdot k \\ 11 \cdot k \end{array}$$

$$k(k - 11) = 0$$

$$k = 0$$

$$\begin{array}{r} k - 11 = 0 \\ +11 \quad +11 \end{array}$$

$$k = 11$$

$$\begin{array}{r} 2 \overline{)12} \\ 2 \overline{)6} \\ 3 \end{array}$$

Example 4: Solve $5g + 12g^2 = 0$.

$$\begin{array}{r} 5 \cdot g \\ 2 \cdot 2 \cdot 3 \cdot g \cdot g \end{array}$$

$$g(5 + 12g) = 0$$

$$g = 0$$

$$\begin{array}{r} 5 + 12g = 0 \\ -5 \quad -5 \end{array}$$

$$\begin{array}{r} 12g = -5 \\ 12 \quad 12 \end{array}$$

$$g = -\frac{5}{12}$$

Example 5: Solve $4y^2 = 10y$.

$$\begin{array}{r} -10y \quad -10y \\ 4y^2 - 10y = 0 \end{array}$$

$$\begin{array}{r} 2 \cdot 2 \cdot y \cdot y \\ 2 \cdot 5 \cdot y \end{array}$$

$$2y(2y - 5) = 0$$

$$\begin{array}{r} 2y = 0 \\ 2 \quad 2 \end{array}$$

$$y = 0$$

$$\begin{array}{r} 2y - 5 = 0 \\ +5 \quad +5 \end{array}$$

$$\frac{2y}{2} = \frac{5}{2}$$

$$y = \frac{5}{2}$$

Example 6: Solve $-10n^2 = 35n$.

$$\begin{array}{r} -35n \quad -35n \\ -10n^2 - 35n = 0 \end{array}$$

$$\begin{array}{r} -2 \cdot 5 \cdot n \cdot n \\ 5 \cdot 7 \cdot n \end{array}$$

$$5n(-2n - 7) = 0$$

$$\begin{array}{r} 5n = 0 \\ 5 \quad 5 \end{array}$$

$$n = 0$$

$$\begin{array}{r} -2n - 7 = 0 \\ +7 \quad +7 \end{array}$$

$$\frac{-2n}{-2} = \frac{7}{-2}$$

$$n = -\frac{7}{2}$$

Example 7: Find the zeros of the function $f(x) = -2x^2 + x$.

set equation
= 0

$$\frac{-2 \cdot x \cdot x}{1 \cdot x}$$

$$\begin{aligned} \downarrow \\ 0 &= -2x^2 + x \\ 0 &= x(-2x + 1) \end{aligned}$$

$$\begin{aligned} \boxed{x=0} \quad \begin{array}{r} -2x + 1 = 0 \\ + 1 -1 \\ \hline -2x = -1 \\ -2 -2 \\ \hline x = \frac{1}{2} \end{array} \end{aligned}$$

Example 8: Find the zeros of the function $f(x) = 12x + 3x^2$.

$$\frac{2 \cdot 2 \cdot 3 \cdot x}{3 \cdot x \cdot x}$$

$$\begin{aligned} \downarrow \\ 0 &= 12x + 3x^2 \\ 0 &= 3x(4 + x) \end{aligned}$$

$$\begin{aligned} \frac{3x=0}{3 \quad 3} \quad \frac{4+x=0}{ + 4 -4} \\ \boxed{x=0} \quad \boxed{x=-4} \end{aligned}$$

Vertical Motion Model

The height h (in feet) of a projectile can be modeled by

$$h = -16t^2 + vt + s$$

where t is the time (in seconds) the object has been in the air,

v is the initial velocity (in feet per second),

and s is the initial height (in feet).

ending
beginning

↳ starting on ground → $s = 0$

Example 9: A startled armadillo jumps straight into the air with an initial velocity of 14 feet per second. After how many seconds does it land on the ground?

$$h = -16t^2 + vt + s$$

$$h = -16t^2 + 14t + 0$$

$$0 = -16t^2 + 14t$$

$$= \frac{2 \cdot 2 \cdot 2 \cdot t \cdot t}{2 \cdot 7 \cdot t}$$

$$0 = \underline{2t}(\underline{-8t} + \underline{7})$$

Example 10: A fountain sprays water into the air with an initial velocity of 20 feet per second.

a) What is the height of the water after half a second?

b) When will the water land on the ground?