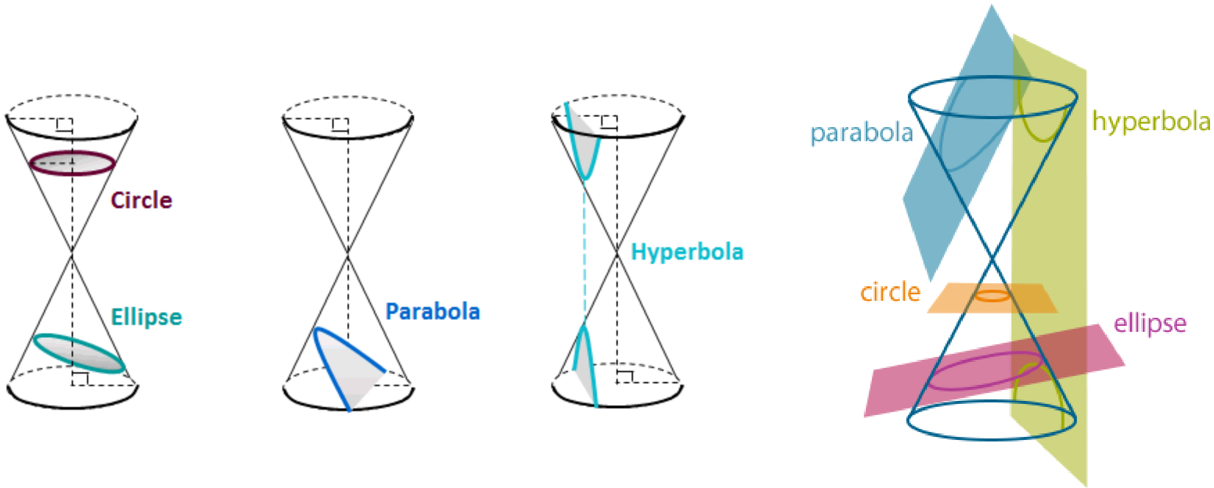
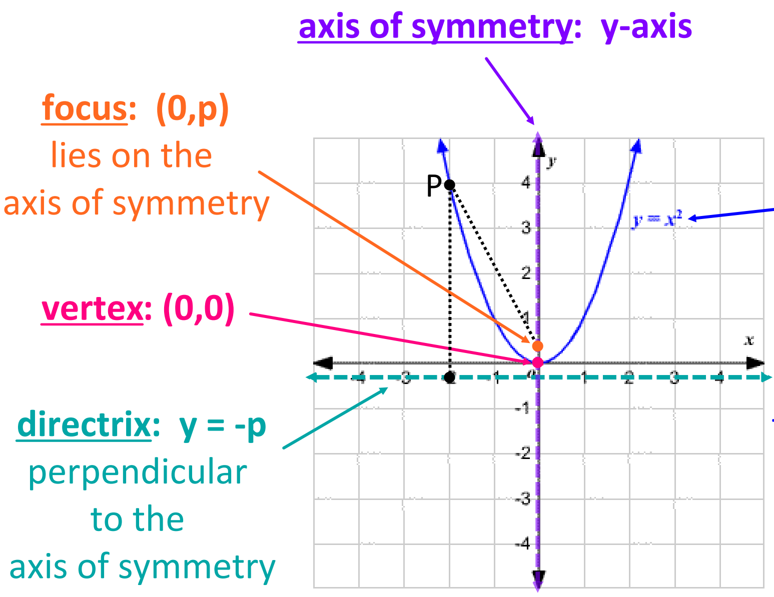
9.1 Introduction to Conic Sections

*The intersection of a double cone and a plane is called a **conic section**.*



9.2 Part 1 Parabolas



STANDARD EQUATION OF A PARABOLA WITH VERTEX (0,0)

$$y = \frac{1}{4p}x^2$$

$p > 0$: opens up

$p < 0$: opens down

The distance between the **vertex and focus** and **vertex and directrix** is the same. This distance is called p.

Definition of Parabola

A **parabola** is the set of all points P(x,y) in the plane whose distance to a fixed point, called the **focus**, equals its distance to a fixed line, called the **directrix**.

Example 1

opens down

Graph $y = -\frac{1}{12}x^2$. Label the vertex, focus, and directrix.

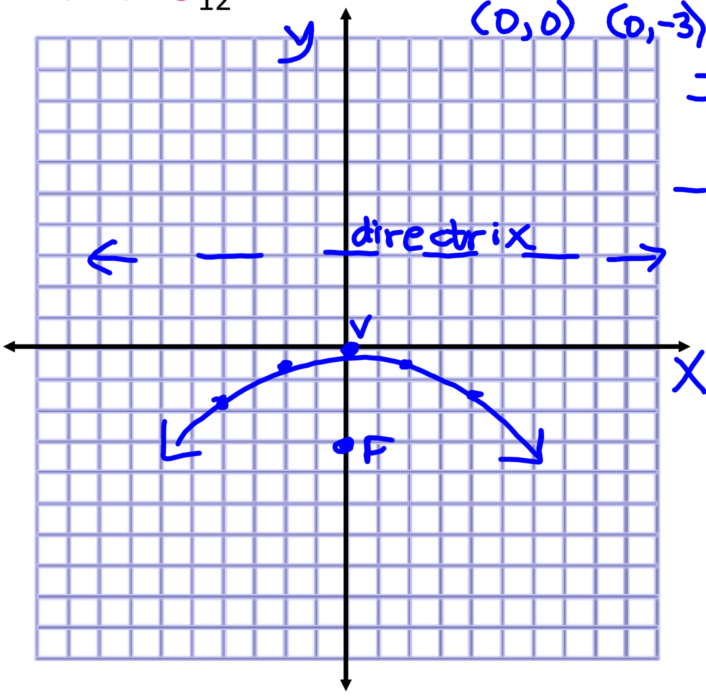
$y = \frac{1}{4p}x^2$

(0,0) (0,-3) y=3

$\frac{-1}{12} = \frac{1}{4p}$

$-4p = 12$

$p = -3$



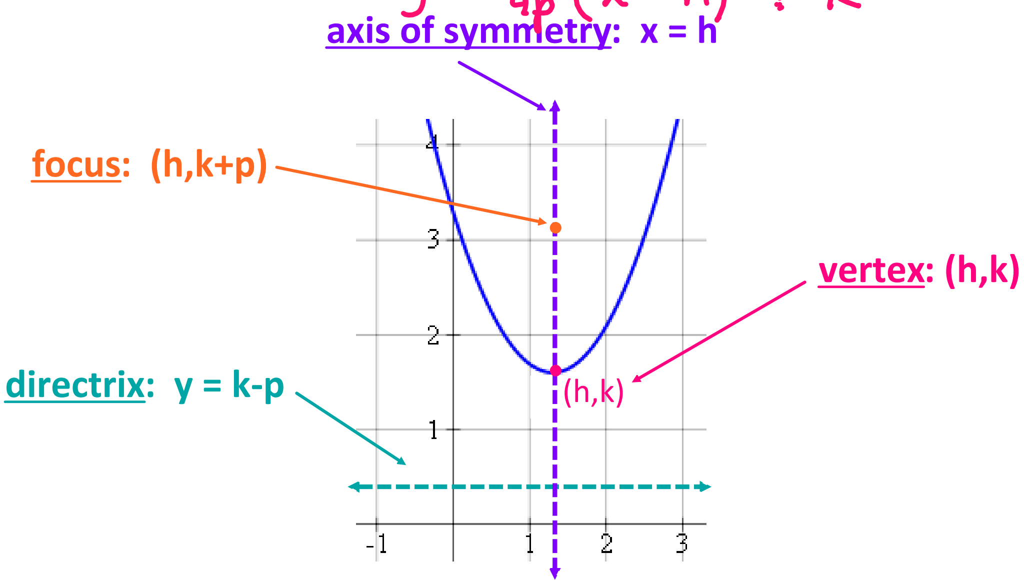
x	y
4	$-\frac{16}{12} = -\frac{4}{3}$
2	$-\frac{4}{12} = -\frac{1}{3}$
0	0
-2	$-\frac{4}{12} = -\frac{1}{3}$
-4	$-\frac{16}{12} = -\frac{4}{3}$

Standard Equation of a Translated Parabola

(meaning its vertex is not at the origin)

$$y - k = \frac{1}{4p}(x - h)^2$$

$$y = \frac{1}{4p}(x-h)^2 + k$$



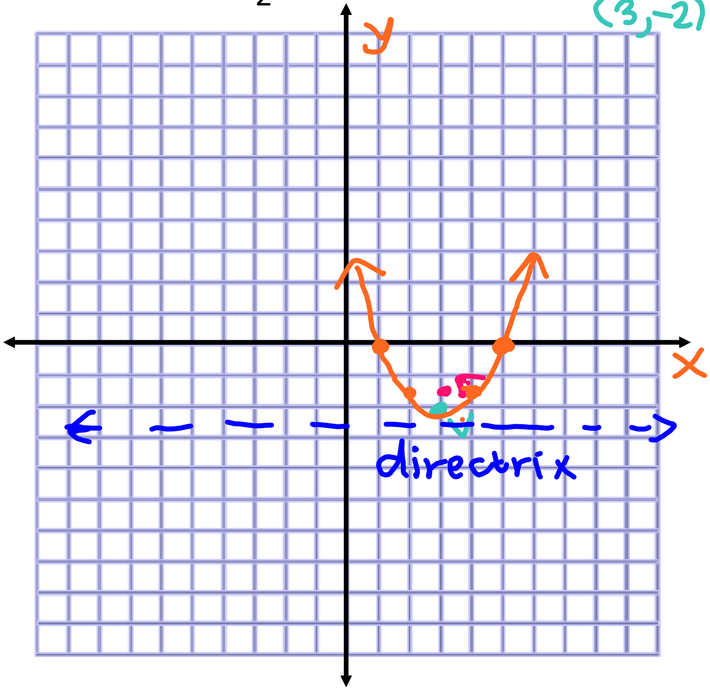
Example 2

$y = \frac{1}{2}(x-3)^2 - 2$

$h=3$
$k=-2$

Graph $y + 2 = \frac{1}{2}(x - 3)^2$. Label the vertex, focus, and directrix.

$(3,-2)$ $(3,-1\frac{1}{2})$ $y = -2\frac{1}{2}$



$\frac{1}{2} = \frac{1}{4p}$

$\frac{2}{4} = \frac{4p}{4}$

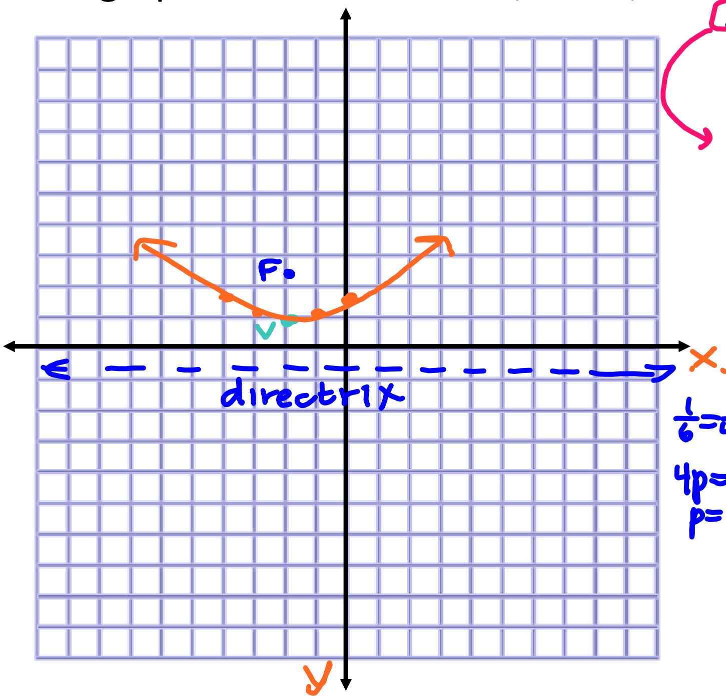
$\frac{1}{2} = p$

x	y
5	0
4	$-1\frac{1}{2}$
3	-2
2	$-1\frac{1}{2}$
1	0

Example 3

comp. the square

Rewrite the equation $x^2 + 4x - 6y = -10$ in standard (vertex) form. Then graph. Label the vertex, focus, and directrix.



$\boxed{x^2 + 4x + 4} = 6y - 10 + 4$

$\frac{1}{2}(4) = 2$

$(2)^2 = 4$

$(x+2)^2 = 6y - 6$

$\frac{(x+2)^2}{6} = \frac{6(y-1)}{6}$

$\frac{1}{6}(x+2)^2 = y - 1$

$y = \frac{1}{6}(x+2)^2 + 1$

$h=-2$ $k=1$ vertex $(-2,1)$

$\frac{1}{6} = \frac{1}{4p}$

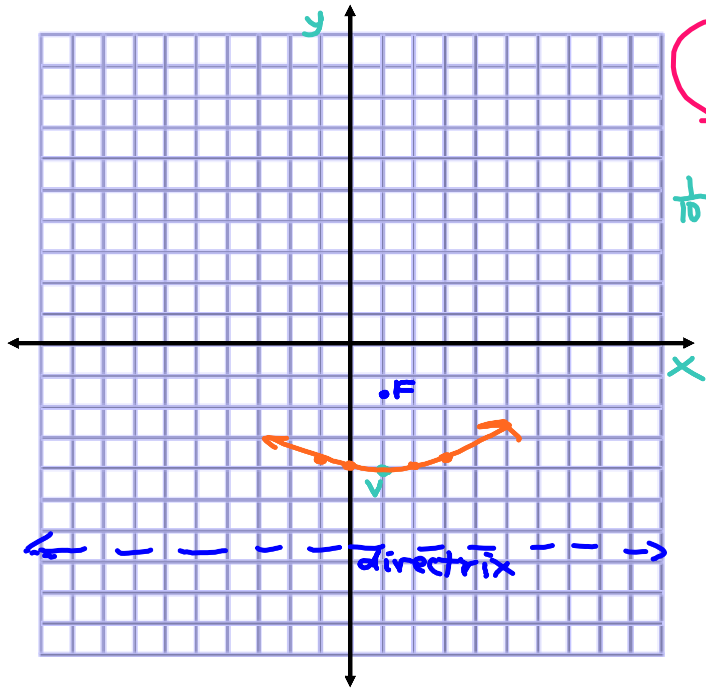
$4p = 6$

$p = \frac{3}{2}$

x	y
0	$1\frac{2}{3}$
-1	$1\frac{1}{6}$
-2	1
-3	$1\frac{1}{6}$
-4	$1\frac{2}{3}$

Example 4

Rewrite the equation $x^2 - 2x - 10y = 39$ in standard (vertex) form. Then graph. Label the vertex, focus, and directrix.



$\boxed{x^2-2x+1} = 10y + 39 + 1$

$\frac{1}{2}(-2) = -1$

$(-1)^2 = 1$

$(x-1)^2 = 10y+40$

$\frac{1}{10}\cdot(x-1)^2 = 10(y+4)\cdot\frac{1}{10}$

$\frac{1}{10}(x-1)^2 = y+4$

$y=\frac{1}{10}(x-1)^2 - 4$

$h=1$
$k=-4$
vertex $(1,-4)$

$\frac{1}{10}=\frac{1}{4p}$

$\frac{4p}{4}=\frac{10}{4}$

$p=\frac{5}{2}$

x	y
3	$-3\frac{3}{5}$
2	$-3\frac{9}{10}$
1	-4
0	$-3\frac{9}{10}$
-1	$-3\frac{3}{5}$

Example 5

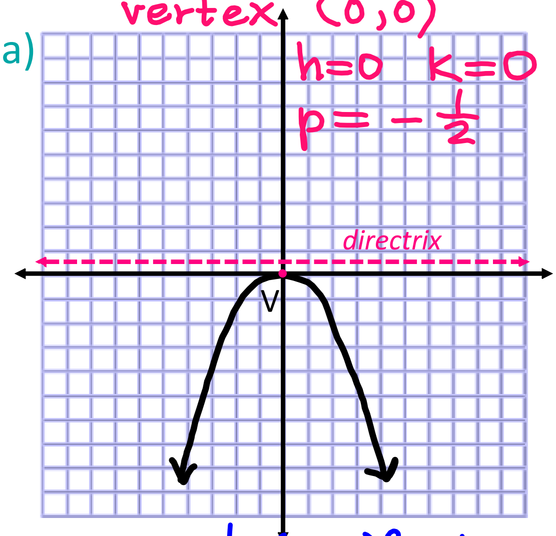
Write the standard equation for each parabola graphed.

a)

vertex $(0,0)$

$h=0 \quad k=0$

$p=-\frac{1}{2}$



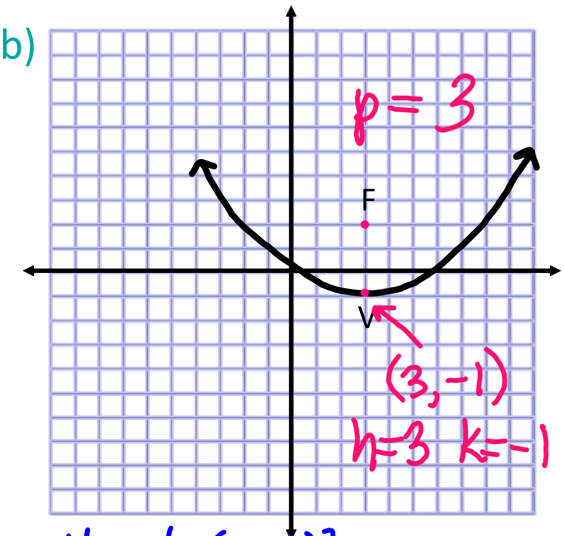
$y=\frac{1}{4p}(x-h)^2+k$

$y=\frac{1}{4(-\frac{1}{2})}(x-0)^2+0$

$\boxed{y=\frac{1}{-2}x^2}$

b)

$p=3$



$(3,-1)$

$h=3 \quad k=-1$

$y=\frac{1}{4p}(x-h)^2+k$

$y=\frac{1}{4(3)}(x-3)^2-1$

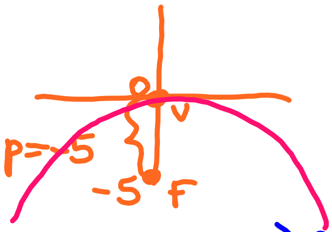
$\boxed{y=\frac{1}{12}(x-3)^2-1}$

Example 6

Write the standard equation for each parabola with the given characteristics.

a) vertex: (0,0) — h k

focus: (0,-5)

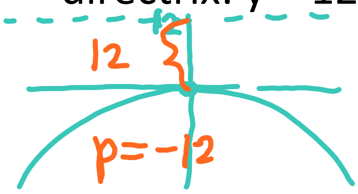


$y=\frac{1}{-20}(x-0)^2+0$

$\boxed{y=-\frac{1}{20}x^2}$

b) vertex: (0,0) — h k

directrix: y = 12

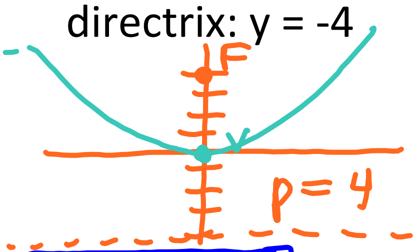


$\boxed{y=\frac{1}{-48}x^2}$

c) focus: (0,4)

directrix: y = -4

h=0 k=0



$\boxed{y=\frac{1}{16}x^2}$