

8.4 Part 1 Greatest Common Factor & The Distributive Property

When two or more numbers are multiplied, each number is a **factor** of the product.

Example 1: Name the factors of 24.

1, 2, 3, 4, 6, 8, 12, 24

Numbers that have only two factors, 1 and itself, are called **prime numbers**. They are whole numbers that are greater than 1.

Example 2: Name the first ten prime numbers.

2, 3, 5, 7, 11, 13, 17, 19, 23, 29

Whole numbers greater than 1 that are not prime are **composite**.

When a whole number is expressed as a product of factors that are all prime, the expression is called the **prime factorization** of the number.

Example: The prime factorization of 18 is $2 \cdot 3 \cdot 3$.

$$\begin{array}{r} 2 \overline{)18} \\ 3 \overline{)9} \\ 3 \end{array}$$

Example 3: Factor each monomial.

a) 200

$$2 \cdot 2 \cdot 2 \cdot 5 \cdot 5$$

$$\begin{array}{r} 2 \overline{)200} \\ 2 \overline{)100} \\ 2 \overline{)50} \\ 5 \overline{)25} \\ 5 \end{array}$$

b) 168

$$2 \cdot 2 \cdot 2 \cdot 3 \cdot 7$$

$$\begin{array}{r} 2 \overline{)168} \\ 2 \overline{)84} \\ 2 \overline{)42} \\ 3 \overline{)21} \\ 7 \end{array}$$

c) $45x^2y^3$

$$3 \cdot 3 \cdot 5 \cdot x \cdot x \cdot y \cdot y \cdot y$$

$$\begin{array}{r} 5 \overline{)45} \\ 3 \overline{)9} \\ 3 \end{array}$$

The **greatest common factor (GCF)** of two or more monomials is the product of their common factors.

Example 4: Find the GCF of the following.

a) 64 and 80

$$\begin{array}{l} 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \\ 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 \end{array}$$

$$\begin{array}{r} 2 \overline{)64} \\ 2 \overline{)32} \\ 2 \overline{)16} \\ 2 \overline{)8} \\ 2 \overline{)4} \\ 2 \end{array}$$

$$\text{GCF} = 2 \cdot 2 \cdot 2 \cdot 2$$

$$\boxed{\text{GCF} = 16}$$

b) $40a^2b$ and $96ab^3$

$$\begin{array}{l} 2 \cdot 2 \cdot 2 \cdot 5 \cdot a \cdot b \\ 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot a \cdot b \cdot b \cdot b \end{array}$$

$$\begin{array}{r} 2 \overline{)40} \\ 2 \overline{)20} \\ 2 \overline{)10} \\ 5 \end{array}$$

$$\begin{array}{r} 2 \overline{)96} \\ 2 \overline{)48} \\ 2 \overline{)24} \\ 2 \overline{)12} \\ 2 \overline{)6} \\ 3 \end{array}$$

$$\text{GCF} = 2 \cdot 2 \cdot 2 \cdot a \cdot b$$

$$\boxed{\text{GCF} = 8ab}$$

Example 5: Find the GCF of the following.

a) $24x^2y^3z$ and $84xy^2z^3$

$$\begin{array}{l} 2 \cdot 2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot y \cdot y \cdot y \cdot z \\ 2 \cdot 2 \cdot 3 \cdot 7 \cdot x \cdot y \cdot y \cdot z \cdot z \cdot z \end{array}$$

$$\begin{array}{r} 2 \overline{)24} \\ 2 \overline{)12} \\ 2 \overline{)6} \\ 3 \end{array}$$

$$\begin{array}{r} 2 \overline{)84} \\ 2 \overline{)42} \\ 3 \overline{)21} \\ 7 \end{array}$$

$$\text{GCF} = 2 \cdot 2 \cdot 3 \cdot x \cdot y \cdot y \cdot z$$

$$\boxed{\text{GCF} = 12xy^2z}$$

b) 54, 72, 108

$$\begin{array}{l} 2 \cdot 3 \cdot 3 \cdot 3 \\ 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \\ 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \end{array}$$

$$\begin{array}{r} 2 \overline{)54} \\ 3 \overline{)27} \\ 3 \overline{)9} \\ 3 \end{array}$$

$$\begin{array}{r} 2 \overline{)72} \\ 2 \overline{)36} \\ 2 \overline{)18} \\ 3 \overline{)9} \\ 3 \end{array}$$

$$\begin{array}{r} 2 \overline{)108} \\ 2 \overline{)54} \\ 3 \overline{)27} \\ 3 \overline{)9} \\ 3 \end{array}$$

$$\text{GCF} = 2 \cdot 3 \cdot 3$$

$$\boxed{\text{GCF} = 18}$$

Factoring Using the Distributive Property

Earlier in Chapter 8 we learned to use the distributive property to multiply a monomial and a polynomial.

Now we will work backwards to put polynomials in factored form.

Example 6: Use the distributive property to factor $10y^2 + 15y$.

$$\boxed{5y(2y + 3)}$$

↑
GCF

$$\begin{array}{r} 2 \overline{)10} \\ 5 \end{array}$$

$$\begin{array}{r} 3 \overline{)15} \\ 5 \end{array}$$

$$\begin{array}{c} 2 \cdot 5 \cdot y \cdot y \\ 3 \cdot 5 \cdot y \\ \hline \text{GCF} = 5y \end{array}$$

Example 7: Use the distributive property to factor

$$18x^2 - 12x^3$$

$$\begin{array}{c} 2 \cdot 3 \cdot 3 \cdot x \cdot x \\ 2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot x \end{array}$$

GCF = $2 \cdot 3 \cdot x \cdot x = 6x^2$

$$\begin{array}{r} 2 \overline{)18} \\ 3 \overline{)9} \\ 3 \end{array}$$

$$\begin{array}{r} 2 \overline{)12} \\ 2 \overline{)6} \\ 3 \end{array}$$

$$\boxed{6x^2(3 - 2x)}$$

Example 8: Use the distributive property to factor

$$28a^4 - 42a^2$$

$$\begin{array}{c} 2 \cdot 2 \cdot 7 \cdot a \cdot a \cdot a \cdot a \\ 2 \cdot 3 \cdot 7 \cdot a \cdot a \end{array}$$

GCF = $2 \cdot 7 \cdot a \cdot a = 14a^2$

$$\begin{array}{r} 2 \overline{)28} \\ 2 \overline{)14} \\ 7 \end{array}$$

$$\begin{array}{r} 2 \overline{)42} \\ 3 \overline{)21} \\ 7 \end{array}$$

$$\boxed{14a^2(2a^2 - 3)}$$

Example 9: Factor $28a^2b + 56abc^2$.

$$\begin{array}{r} 2 \overline{) 28} \\ 2 \overline{) 14} \\ \underline{7} \end{array}$$

$$\begin{array}{r} 2 \overline{) 56} \\ 2 \overline{) 28} \\ \underline{2 \overline{) 14}} \\ \underline{7} \end{array}$$

$$\begin{array}{l} 2 \cdot 2 \cdot 7 \cdot a \cdot a \cdot b \\ 2 \cdot 2 \cdot 2 \cdot 7 \cdot a \cdot b \cdot c \cdot c \end{array}$$

$$\text{GCF} = 2 \cdot 2 \cdot 7 \cdot a \cdot b = 28ab$$

$$28ab(a + 2c^2)$$

Example 10: Factor $20x^3y - 24x^2y^3$.

Example 11: Factor $16fg^2 - 24g^2h + 40g^2$.

$$\begin{array}{r} 2 \overline{) 16} \\ 2 \overline{) 8} \\ 2 \overline{) 4} \\ \underline{2} \end{array}$$

$$\begin{array}{r} 2 \overline{) 24} \\ 2 \overline{) 12} \\ 2 \overline{) 6} \\ \underline{3} \end{array}$$

$$\begin{array}{r} 2 \overline{) 40} \\ 2 \overline{) 20} \\ 2 \overline{) 10} \\ \underline{5} \end{array}$$

$$\begin{array}{l} 2 \cdot 2 \cdot 2 \cdot 2 \cdot f \cdot g \cdot g \\ 2 \cdot 2 \cdot 2 \cdot 3 \cdot g \cdot g \cdot h \\ 2 \cdot 2 \cdot 2 \cdot 5 \cdot g \cdot g \end{array}$$

$$\text{GCF} = 2 \cdot 2 \cdot 2 \cdot g \cdot g = 8g^2$$

$$8g^2(2f - 3h + 5)$$

Example 12: Factor $6k^3m + 14k^2m^2 - 4k^3m^3$.

$$\begin{array}{r} 2 \overline{) 6} \\ \underline{3} \end{array} \quad \begin{array}{r} 2 \overline{) 14} \\ \underline{7} \end{array} \quad \begin{array}{r} 2 \overline{) 4} \\ \underline{2} \end{array}$$

$$\begin{array}{l} 2 \cdot 3 \cdot k \cdot k \cdot k \cdot m \\ 2 \cdot 7 \cdot k \cdot k \cdot m \cdot m \\ 2 \cdot 2 \cdot k \cdot k \cdot k \cdot m \cdot m \cdot m \end{array}$$

$$\text{GCF} = 2 \cdot k \cdot k \cdot m = 2k^2m$$

$$2k^2m(3k + 7m - 2km^2)$$