8.4 Part 1 Greatest Common Factor & The Distributive Property

When two or more numbers are multiplied,
each number is a factor of the product.

Example 1: Name the factors of 24.

1, 2, 3, 4, 6, 8, 12, 24

Numbers that have only two factors, 1 and itself, are called prime numbers. They are whole numbers that are greater than 1.

Example 2: Name the first ten prime numbers.

2, 3, 5, 7, 11, 13, 17, 19, 23, 29

Whole numbers greater than 1 that are not prime are composite.

When a whole number is expressed as a product of factors that are all prime, the expression is called the prime factorization of the number.

Example: The prime factorization of 18 is $2 \cdot 3 \cdot 3$.

2 | 18
3 | 9
3

Example 3: Factor each monomial.

a) 200

$2 \cdot 2 \cdot 2 \cdot 5 \cdot 5$

2 | 200
2 | 100
2 | 50
5 | 25
5

b) 168

$2 \cdot 2 \cdot 2 \cdot 3 \cdot 7$

2 | 168
2 | 84
2 | 42
3 | 21
7

c) $45x^2y^3$

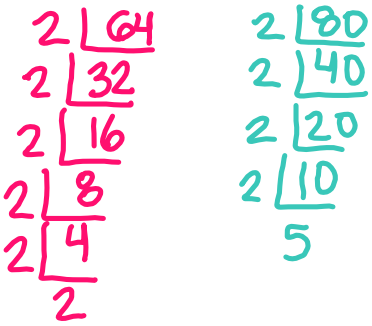
$3 \cdot 3 \cdot 5 \cdot x \cdot x \cdot y \cdot y \cdot y$

5 | 45
3 | 9
3

The greatest common factor (GCF) of two or more monomials is the product of their common factors.

Example 4: Find the GCF of the following.

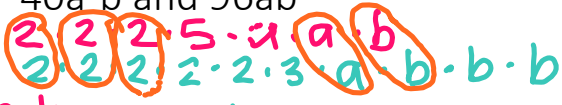
a) 64 and 80

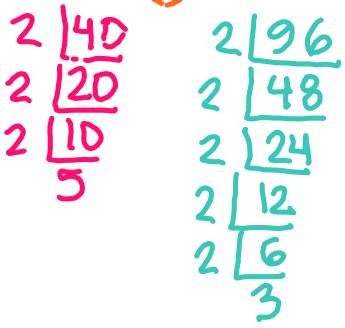


$GCF = 2 \cdot 2 \cdot 2 \cdot 2$

$GCF = 16$

b) $40a^2b$ and $96ab^3$



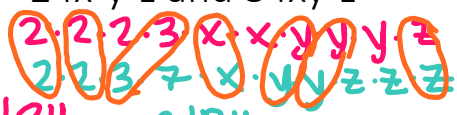


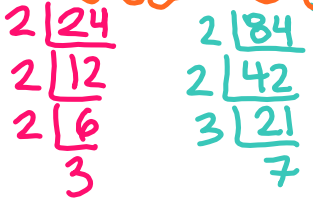
$GCF = 2 \cdot 2 \cdot 2 \cdot a \cdot b$

$GCF = 8ab$

Example 5: Find the GCF of the following.

a) $24x^2y^3z$ and $84xy^2z^3$

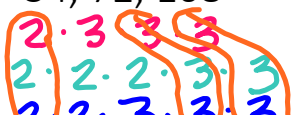


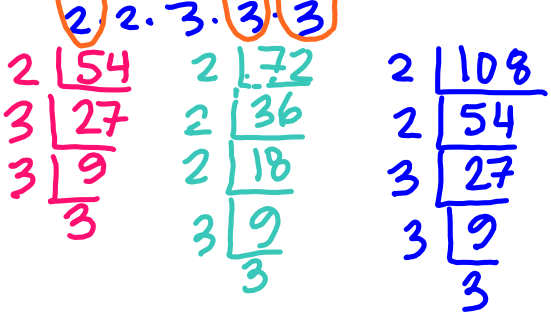


$GCF = 2 \cdot 2 \cdot 3 \cdot x \cdot y \cdot y \cdot z$

$GCF = 12xy^2z$

b) 54, 72, 108





$GCF = 2 \cdot 3 \cdot 3$

$GCF = 18$

Factoring Using the Distributive Property

Earlier in Chapter 8 we learned to use the distributive property to multiply a monomial and a polynomial.

Now we will work backwards to put polynomials in factored form.

Example 6: Use the distributive property to factor $10y^2 + 15y$.

$5y(2y+3)$

GCF

$2 \cdot 5 \cdot y \cdot y$

$3 \cdot 5 \cdot y$

GCF = 5y

$2 \mid 10$ 5 $\quad 3 \mid 15$ 5

Example 7: Use the distributive property to factor $18x^2 - 12x^3$.

$2 \cdot 3 \cdot 3 \cdot x \cdot x$

$2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot x$

GCF = $2 \cdot 3 \cdot x \cdot x = 6x^2$

$2 \mid 18$ $3 \mid 9$ 3 $\quad 2 \mid 12$ $2 \mid 6$ 3

$6x^2(3 - 2x)$

Example 8: Use the distributive property to factor $28a^4 - 42a^2$.

$2 \cdot 2 \cdot 7 \cdot a \cdot a \cdot a \cdot a$

$2 \cdot 3 \cdot 7 \cdot a \cdot a$

GCF = $2 \cdot 7 \cdot a \cdot a = 14a^2$

$2 \mid 28$ $2 \mid 14$ 7 $\quad 2 \mid 42$ $3 \mid 21$ 7

$14a^2(2a^2 - 3)$

Example 9: Factor $28a^2b + 56abc^2$.

$2 \mid 28$, $2 \mid 14$, 7

$2 \mid 56$, $2 \mid 28$, $2 \mid 14$, 7

$2 \cdot 2 \cdot 7 \cdot a \cdot a \cdot b$

$2 \cdot 2 \cdot 2 \cdot 7 \cdot a \cdot b \cdot c \cdot c$

$GCF = 2 \cdot 2 \cdot 7 \cdot a \cdot b = 28ab$

$28ab(a + 2c^2)$

Example 10: Factor $20x^3y - 24x^2y^3$.

Example 11: Factor $16fg^2 - 24g^2h + 40g^2$.

$2 \mid 16$, $2 \mid 8$, $2 \mid 4$, 2

$2 \mid 24$, $2 \mid 12$, $2 \mid 6$, 3

$2 \mid 40$, $2 \mid 20$, $2 \mid 10$, 5

$2 \cdot 2 \cdot 2 \cdot 2 \cdot f \cdot g \cdot g$

$2 \cdot 2 \cdot 2 \cdot 3 \cdot g \cdot g \cdot h$

$2 \cdot 2 \cdot 2 \cdot 5 \cdot g \cdot g$

$GCF = 2 \cdot 2 \cdot 2 \cdot g \cdot g = 8g^2$

$8g^2(2f - 3h + 5)$

Example 12: Factor $6k^3m + 14k^2m^2 - 4k^3m^3$.

$2 \mid 6$, 3

$2 \mid 14$, 7

$2 \mid 4$, 2

$2 \cdot 3 \cdot k \cdot k \cdot k \cdot m$

$2 \cdot 7 \cdot k \cdot k \cdot m \cdot m$

$2 \cdot 2 \cdot k \cdot k \cdot k \cdot m \cdot m \cdot m$

$GCF = 2 \cdot k \cdot k \cdot m = 2k^2m$

$2k^2m(3k + 7m - 2km^2)$