OTHER ANGLE RELATIONSHIPS IN CIRCLES

The Luis I bridge in Oporto, Portugal was designed by T. Seyrig and completed in 1885.

What does the upper road represent? **tangent**

What does the lower road represent? **secant**

**Theorem 10.12**

If a tangent and a secant/chord intersect **on the circle**, then the measure of each angle formed is one-half the measure of its intercepted arc.

\[
m \angle TCA = \frac{1}{2} m \widehat{CA} \\
m \angle SCA = \frac{1}{2} m \widehat{CBA}
\]
**Theorem 10.13**
If two chords/secants intersect in the interior of a circle, then the measure of an angle formed is one-half the sum of the measures of the arcs intercepted by the angle and its vertical angle.

\[
m \angle 1 = \frac{1}{2} (mRU + mST)
\]
\[
m \angle 3 = \frac{1}{2} (mRU + mST)
\]
\[
m \angle 2 = \frac{1}{2} (mRS + mUT)
\]
\[
m \angle 4 = \frac{1}{2} (mRS + mUT)
\]

**Theorem 10.14**
If two secants, a secant and a tangent, or two tangents intersect in the exterior of a circle, then the measure of the angle formed is one-half the positive difference of the measures of the intercepted arcs.

**Case 1:** two secants

\[
m \angle CAT = \frac{1}{2} (mCT - mBR)
\]

**Case 2:** a secant & a tangent

\[
m \angle FDG = \frac{1}{2} (mFG - mEG)
\]

**Case 3:** two tangents

\[
m \angle IHG = \frac{1}{2} (mIX - mI)
\]
SUMMARY

- ON THE CIRCLE = $\frac{1}{2} \text{arc}$

- IN THE CIRCLE = $\frac{1}{2} (\text{arc} \ 1 + \text{arc} \ 2)$

- OUTSIDE THE CIRCLE = $\frac{1}{2} (\text{big arc} - \text{little arc})$

Example 1
Line $m$ is tangent to the circle.

a) Find $\angle PRT$.
   \[
   \angle = \frac{1}{2} \cdot 152° = 76°
   \]

b) Find $\angle RST$.
   \[
   360° - 152° = 208°
   \]
Example 2

a) Find $m \angle DKB$.

\[
\text{angle} = \frac{1}{2} (\text{arc 1} + \text{arc 2}) \\
= \frac{1}{2} (80 + 60) \\
= \frac{1}{2} (140) \\
\text{angle} = 70^\circ
\]

b) Find $m \angle AKD$.

\[
70^\circ + \text{angle} = 180^\circ \\
\text{angle} = 110^\circ
\]

Example 3

Find $m \angle DBR$.

\[
\text{angle} = \frac{1}{2} (\text{big arc} - \text{little arc}) \\
\text{angle} = \frac{1}{2} (190 - 60) \\
\text{angle} = \frac{1}{2} (130^\circ) \\
\text{angle} = 65^\circ
\]
**Example 4**

If $m_{BE} = 100$ and $m_{AR} = 25$,

a) Find $m \angle 1$.

\[
\angle = \frac{1}{2}(\text{big} - \text{little})
\]

\[
m_{\angle 1} = \frac{1}{2}(100 - 25)
\]

\[
m_{\angle 1} = 37.5^\circ
\]

b) Find $m \angle 2$.

\[
\angle = \frac{1}{2}\text{arc}
\]

\[
m_{\angle 2} = \frac{1}{2}(25)
\]

\[
m_{\angle 2} = 12.5^\circ
\]

c) Find $m \angle 3$.

\[
\angle = \frac{1}{2}\text{arc}
\]

\[
m_{\angle 3} = \frac{1}{2}(100)
\]

\[
m_{\angle 3} = 50^\circ
\]

**Example 5**

If $m_{AB} = 120$, $m_{BC} = 30$, and $m_{CD} = 160$, find $m \angle Q$.

\[
\angle = \frac{1}{2}(\text{big} - \text{little})
\]

\[
m_{\angle Q} = \frac{1}{2}(160 - 30)
\]

\[
m_{\angle Q} = 65^\circ
\]

\[
m_{\angle Q} = 10^\circ
\]
**Example 6**

Use Circle K to find the value of x.

\[
\text{angle} = \frac{1}{2} (\text{big} - \text{little})
\]

\[
x + 2.5 = \frac{1}{2} (4x + 5 - 50)
\]

\[
x + 2.5 = \frac{1}{2} (4x - 45)
\]

\[
x + 2.5 = 2x - 22.5
\]

\[
-x
\]

\[
2.5 = x - 22.5
\]

\[
\frac{2.5}{x} = 22.5
\]

\[
25 = x
\]

**Example 7**

Use Circle S to find the value of y.

\[
\text{angle} = \frac{1}{2} (\text{big} - \text{little})
\]

\[
28 = \frac{1}{2} (360 - y - y)
\]

\[
28 = \frac{1}{2} (360 - 2y)
\]

\[
28 = 180 - y
\]

\[
-152 = -180
\]

\[
152 = y
\]