

8.4 Greatest Common Factor & The Zero-Product Property

When two or more numbers are multiplied, each number is a **factor** of the product.

Example 1: Name the factors of 24.

1, 2, 3, 4, 6, 8, 12, 24

Numbers that have only two factors, 1 and itself, are called **prime numbers**. They are whole numbers that are greater than 1.

Example 2: Name the first ten prime numbers.

2, 3, 5, 7, 11, 13, 17, 19, 23, 29

Whole numbers greater than 1 that are not prime are **composite**.

When a whole number is expressed as a product of factors that are all prime, the expression is called the **prime factorization** of the number.

Example: The prime factorization of 18 is $2 \cdot 3 \cdot 3$.

$$\begin{array}{r} 2 \overline{)18} \\ 3 \overline{)9} \\ 3 \end{array}$$

Example 3: Factor each monomial.

a) 200 $2^3 \cdot 5^2$

$$2 \cdot 2 \cdot 2 \cdot 5 \cdot 5$$

$$2 \overline{)200}$$

$$2 \overline{)100}$$

$$2 \overline{)50}$$

$$5 \overline{)25}$$

$$5$$

b) 168 $2^3 \cdot 3 \cdot 7$

$$2 \cdot 2 \cdot 2 \cdot 3 \cdot 7$$

$$2 \overline{)168}$$

$$2 \overline{)84}$$

$$2 \overline{)42}$$

$$3 \overline{)21}$$

$$7$$

c) $45x^2y^3$

$$3 \cdot 3 \cdot 5 \cdot x \cdot x \cdot y \cdot y \cdot y$$

$$5 \overline{)45}$$

$$3 \overline{)9}$$

$$3$$

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The **greatest common factor (GCF)** of two or more monomials is the product of their common factors.

Example 4: Find the GCF of the following.

a) 64 and 80

$$\text{GCF} = 16$$

$$\begin{array}{r} 2 \overline{)64} \\ 2 \overline{)32} \\ 2 \overline{)16} \\ 2 \overline{)8} \\ 2 \overline{)4} \\ 2 \end{array} \quad \begin{array}{r} 2 \overline{)80} \\ 2 \overline{)40} \\ 2 \overline{)20} \\ 2 \overline{)10} \\ 5 \end{array}$$

$$\begin{array}{c} 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \\ 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 \\ \hline 2 \cdot 2 \cdot 2 \cdot 2 = 16 \end{array}$$

b) $40a^2b$ and $96ab^3$

$$\text{GCF} = 8ab$$

$$\begin{array}{r} 2 \overline{)40} \\ 2 \overline{)20} \\ 2 \overline{)10} \\ 5 \end{array} \quad \begin{array}{r} 2 \overline{)96} \\ 2 \overline{)48} \\ 2 \overline{)24} \\ 2 \overline{)12} \\ 2 \overline{)6} \\ 3 \end{array}$$

$$\begin{array}{c} 2 \cdot 2 \cdot 2 \cdot 5 \cdot a \cdot a \cdot b \\ 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot a \cdot b \cdot b \cdot b \\ \hline 2 \cdot 2 \cdot 2 \cdot a \cdot b = 8ab \end{array}$$

c) 54, 72, 108

$$\text{GCF} = 18$$

$$\begin{array}{r} 2 \overline{)54} \\ 3 \overline{)27} \\ 3 \overline{)9} \\ 3 \end{array} \quad \begin{array}{r} 2 \overline{)72} \\ 2 \overline{)36} \\ 2 \overline{)18} \\ 3 \overline{)9} \\ 3 \end{array} \quad \begin{array}{r} 2 \overline{)108} \\ 2 \overline{)54} \\ 3 \overline{)27} \\ 3 \overline{)9} \\ 3 \end{array}$$

$$\begin{array}{c} 2 \cdot 3 \cdot 3 \cdot 3 \\ 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \\ 2 \cdot 3 \cdot 3 \cdot 3 \\ \hline 2 \cdot 3 \cdot 3 = 18 \end{array}$$

Factoring Using the Distributive Property

Earlier in Chapter 8 we learned to use the distributive property to **multiply a monomial and a polynomial**.

Now we will work backwards to put polynomials in **factored form**.

Example 5: Use the distributive property to factor $10y^2 + 15y$.

$$5y(2y + 3)$$

factored form

$$\begin{array}{r} 2 \overline{)10} \\ 5 \end{array} \quad \begin{array}{r} 3 \overline{)15} \\ 5 \end{array}$$

$$\begin{array}{r} 2 \cdot 5 \cdot y \cdot y \\ 3 \cdot 5 \cdot y \end{array}$$

$$\text{GCF} = 5y$$

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Example 6: Use the distributive property to factor $18x^2 - 12x^3$.

$$6x^2(3 - 2x)$$

$$\begin{array}{r} 3 \overline{)18} \\ 2 \overline{)6} \\ 3 \end{array} \quad \begin{array}{r} 3 \overline{)12} \\ 2 \overline{)4} \\ 2 \end{array}$$

$$\begin{array}{cccc} 2 \cdot 3 \cdot 3 \cdot x \cdot x & & & \\ 2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot x & & & \\ \hline \text{GCF} = 6x^2 & & & \end{array}$$

Example 7: Factor $16fg^2 - 24g^2h + 40g^2$.

$$8g^2(2f - 3h + 5)$$

$$\begin{array}{ccccccc} 2 \cdot 2 \cdot 2 \cdot 2 \cdot f \cdot g \cdot g & & & & & & \\ 2 \cdot 2 \cdot 2 \cdot 3 \cdot g \cdot g \cdot h & & & & & & \\ 2 \cdot 2 \cdot 2 \cdot 5 \cdot g \cdot g & & & & & & \\ \hline \text{GCF} = 8g^2 & & & & & & \end{array}$$

Example 8: Factor $6k^3m + 14k^2m^2 - 4k^3m^3$.

$$2k^2m(3k + 7m - 2km^2)$$

$$\begin{array}{ccccccc} 2 \cdot 3 \cdot k \cdot k \cdot k \cdot m & & & & & & \\ 2 \cdot 7 \cdot k \cdot k \cdot m \cdot m & & & & & & \\ 2 \cdot 2 \cdot k \cdot k \cdot k \cdot m \cdot m \cdot m & & & & & & \\ \hline \text{GCF} = 2k^2m & & & & & & \end{array}$$

Solving Equations by Factoring

Zero Product Property

For all numbers a and b, if $ab = 0$, then $a = 0$, $b = 0$, or both a and b equal 0.

factored form

Example 9: Solve $(y + 2)(3y + 5) = 0$. Then check each solution.

$$\begin{array}{r} y + 2 = 0 \\ -2 \quad -2 \\ \hline \boxed{y = -2} \end{array}$$

$$\begin{array}{r} 3y + 5 = 0 \\ -5 \quad -5 \\ \hline 3y = -5 \\ \frac{3y}{3} = \frac{-5}{3} \\ \boxed{y = -\frac{5}{3}} \end{array}$$

$$\begin{array}{l} y = -2 \\ (-2 + 2)(3 \cdot -2 + 5) \stackrel{?}{=} 0 \\ (0)(-1) = 0 \checkmark \end{array}$$

$$\begin{array}{l} y = -\frac{5}{3} \\ (-\frac{5}{3} + 2)(3 \cdot -\frac{5}{3} + 5) \stackrel{?}{=} 0 \\ (\frac{1}{3})(0) = 0 \checkmark \end{array}$$

Example 10: Solve $(2a + 4)(a - 9) = 0$.

$$\begin{array}{r} 2a + 4 = 0 \\ -4 \quad -4 \\ \hline 2a = -4 \\ \frac{2a}{2} = \frac{-4}{2} \\ \boxed{a = -2} \end{array}$$

$$\begin{array}{r} a - 9 = 0 \\ +9 \quad +9 \\ \hline \boxed{a = 9} \end{array}$$

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Example 11: Solve $k^2 - 11k = 0$. factored form

$$\begin{array}{l} k \cdot k \\ 11 \cdot k \\ \text{GCF} = k \end{array}$$

$$\begin{array}{l} k(k-11) = 0 \\ \boxed{k=0} \quad \begin{array}{l} k-11=0 \\ +11 \quad +11 \\ \hline \boxed{k=11} \end{array} \end{array}$$

Example 12: Solve $-10n^2 = 35n$.

$$\begin{array}{l} +10n^2 \quad +10n^2 \\ \hline 0 = 35n + 10n^2 \\ 0 = 5n(7 + 2n) \\ \begin{array}{l} 5n=0 \\ \frac{5n}{5} = \frac{0}{5} \\ \boxed{n=0} \end{array} \quad \begin{array}{l} 7+2n=0 \\ \frac{7+2n}{-7} = \frac{0}{-7} \\ \frac{2n}{2} = \frac{-7}{2} \\ \boxed{n = -\frac{7}{2}} \end{array} \end{array}$$

$$\begin{array}{l} 5 \cdot 7 \cdot n \\ 5 \cdot 2 \cdot n \cdot n \\ \text{GCF} = 5n \end{array}$$

Example 13: Solve $4y^2 = 10y$.

$$\begin{array}{l} -4y^2 \quad -4y^2 \\ \hline 0 = 10y - 4y^2 \\ 0 = 2y(5 - 2y) \\ \begin{array}{l} 2y=0 \\ \frac{2y}{2} = \frac{0}{2} \\ \boxed{y=0} \end{array} \quad \begin{array}{l} 5-2y=0 \\ \frac{5-2y}{-5} = \frac{0}{-5} \\ \frac{-2y}{-2} = \frac{-5}{-2} \\ \boxed{y = \frac{5}{2}} \end{array} \end{array}$$

$$\begin{array}{l} 5 \cdot 2 \cdot y \\ 2 \cdot 2 \cdot y \cdot y \\ \text{GCF} = 2y \end{array}$$

Example 14: Find the zeros of the function $f(x) = -2x^2 + x$.

$$\begin{array}{l} -2x \cdot x \\ 1 \cdot x \\ \text{GCF} = x \\ \boxed{x=0} \end{array}$$

$$\begin{array}{l} 0 = -2x^2 + x \\ 0 = x(-2x + 1) \\ \begin{array}{l} -2x+1=0 \\ \frac{-2x+1}{-1} = \frac{0}{-1} \\ \frac{-2x}{-2} = \frac{-1}{-2} \\ \boxed{x = \frac{1}{2}} \end{array} \end{array}$$

Vertical Motion Model

The height h (in feet) of a projectile can be modeled by

$$h = -16t^2 + vt + s$$

where t is the time (in seconds) the object has been in the air, v is the initial velocity (in feet per second), and s is the initial height (in feet).

Example 15: A startled armadillo jumps straight into the air with an initial velocity of 14 feet per second. After how many seconds does it land on the ground? $s = 0$ (on the ground)
 $h = 0$ (end on ground)

$$0 = -16t^2 + 14t + 0$$

$$0 = -16t^2 + 14t$$

$$0 = 2t(-8t + 7)$$

$$\frac{2t}{2} = \frac{0}{2}$$

$$t = 0$$

$$\frac{-8t + 7}{7} = \frac{0}{7}$$

$$\frac{-8t}{-8} = \frac{-7}{-8}$$

$$t = \frac{7}{8} \text{ sec}$$

Example 16: A fountain sprays water into the air with an initial velocity of 20 feet per second. $v = 20$ $s = 0$

a) What is the height of the water after half a second?

$$h = ? \quad \text{when } t = \frac{1}{2}$$

$$h = -16t^2 + 20t$$

$$h = -16\left(\frac{1}{2}\right)^2 + 20\left(\frac{1}{2}\right)$$

$$h = 6 \text{ feet}$$

b) When will the water land on the ground?

$$t = ?$$

$$\text{when } h = 0$$

$$0 = -16t^2 + 20t$$

$$0 = 4t(-4t + 5)$$

$$\frac{4t}{4} = \frac{0}{4}$$

$$t = 0$$

$$\frac{-4t + 5}{-5} = \frac{0}{-5}$$

$$\frac{-4t}{-4} = \frac{-5}{-4}$$

$$t = \frac{5}{4} \text{ sec}$$