8.4 Greatest Common Factor & The Zero-Product Property

When two or more numbers are multiplied, each number is a factor of the product.

Example 1: Name the factors of 24.

1, 2, 3, 4, 6, 8, 12, 24

Numbers that have only two factors, 1 and itself, are called prime numbers. They are whole numbers that are greater than 1.

Example 2: Name the first ten prime numbers.

2, 3, 5, 7, 11, 13, 17, 19, 23, 29

Whole numbers greater than 1 that are not prime are composite.

When a whole number is expressed as a product of factors that are all prime, the expression is called the prime factorization of the number.

Example: The prime factorization of 18 is $2 \cdot 3 \cdot 3$.

2 | 18
3 | 9
3

Example 3: Factor each monomial.

a) 200 $2^3 \cdot 5^2$

$2 \cdot 2 \cdot 2 \cdot 5 \cdot 5$

2 | 200
2 | 100
2 | 50
5 | 25
5

b) 168 $2^3 \cdot 3 \cdot 7$

$2 \cdot 2 \cdot 2 \cdot 3 \cdot 7$

2 | 168
2 | 84
2 | 42
3 | 21
7

c) $45x^2y^3$

$3 \cdot 3 \cdot 5 \cdot x \cdot x \cdot y \cdot y \cdot y$

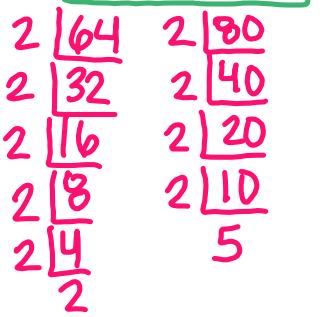
5 | 45
3 | 9
3

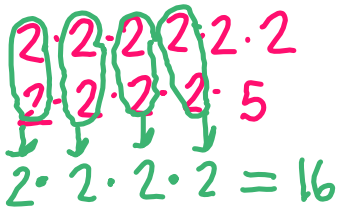
The greatest common factor (GCF) of two or more monomials is the product of their common factors.

Example 4: Find the GCF of the following.

a) 64 and 80

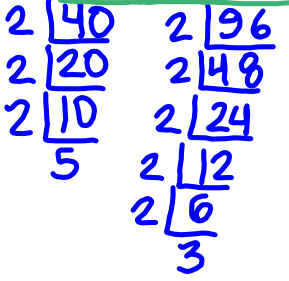
GCF = 16

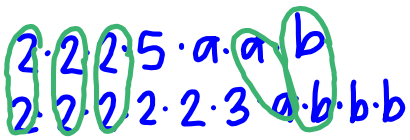




b) $40a^2b$ and $96ab^3$

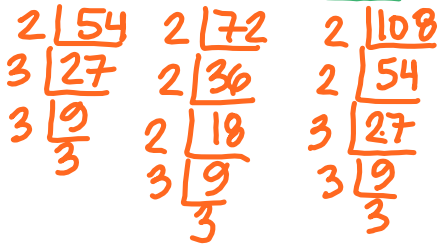
GCF = 8ab

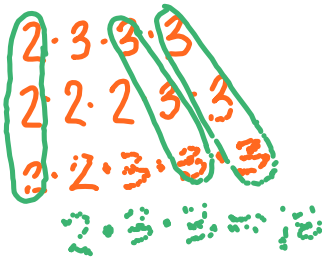




c) 54, 72, 108

GCF = 18





Factoring Using the Distributive Property

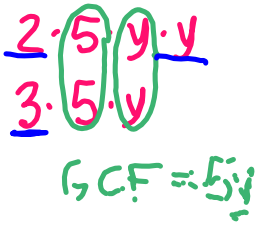
Earlier in Chapter 8 we learned to use the distributive property to multiply a monomial and a polynomial.

Now we will work backwards to put polynomials in factored form.

Example 5: Use the distributive property to factor $10y^2 + 15y$.

$5y(2y + 3)$

factored form



Example 6: Use the distributive property to factor $18x^2 - 12x^3$.

$6x^2(3-2x)$

$18 = 2 \cdot 3 \cdot 3$, $12 = 2 \cdot 2 \cdot 3$

$2 \cdot 3 \cdot 3 \cdot x \cdot x$

$2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot x$

GCF $= 6x^2$

Example 7: Factor $16fg^2 - 24g^2h + 40g^2$.

$8g^2(2f-3h+5)$

$2 \cdot 2 \cdot 2 \cdot 2 \cdot f \cdot g \cdot g$

$2 \cdot 2 \cdot 2 \cdot 3 \cdot g \cdot g \cdot h$

$2 \cdot 2 \cdot 2 \cdot 5 \cdot g \cdot g$

GCF $= 8g^2$

Example 8: Factor $6k^3m + 14k^2m^2 - 4k^3m^3$.

$2k^2m(3k+7m-2km^2)$

$2 \cdot 3 \cdot k \cdot k \cdot k \cdot m$

$2 \cdot 7 \cdot k \cdot k \cdot m \cdot m$

$2 \cdot 2 \cdot k \cdot k \cdot k \cdot m \cdot m \cdot m$

GCF $= 2k^2m$

Solving Equations by Factoring

Zero Product Property

For all numbers a and b, if ab = 0,
then a = 0, b = 0, or both a and b equal 0.

Example 9: Solve (y + 2)(3y + 5) = 0. Then check each solution. (factored form)

$y+2=0$, $-2 \quad -2$

$y=-2$ ✓

$3y+5=0$, $-5 \quad -5$

$\frac{3y}{3} = \frac{-5}{3}$

$y=-\frac{5}{3}$ ✓

$y=-2$: $(-2+2)(3\cdot-2+5) \stackrel{?}{=} 0$

$(0)(-1)=0$ ✓

$y=-\frac{5}{3}$: $(-\frac{5}{3}+2)(3\cdot-\frac{5}{3}+5) \stackrel{?}{=} 0$

$(\frac{1}{3})(0)=0$ ✓

Example 10: Solve (2a + 4)(a - 9)= 0.

$2a+4=0$, $-4 \quad -4$

$\frac{2a}{2} = \frac{-4}{2}$

$a=-2$

$a-9=0$, $+9 \quad +9$

$a=9$

Example 11: Solve $k^2 - 11k = 0$. factored form

$k \cdot k$
$11 \cdot k$
GCF = k

$k(k-11)=0$

$k=0$

$k-11=0$
$+11 \quad +11$
$k=11$

Example 12: Solve $-10n^2 = 35n$.

$+10n^2 \quad +10n^2$
$0 = 35n + 10n^2$
$0 = 5n(7 + 2n)$

$5 \cdot 7 \cdot n$
$5 \cdot 2 \cdot n \cdot n$
GCF = 5n

$\frac{5n}{5} = \frac{0}{5}$
$n=0$

$7+2n=0$
$-7 \quad -7$
$\frac{2n}{2} = \frac{-7}{2}$
$n=-\frac{7}{2}$

Example 13: Solve $4y^2 = 10y$.

$-4y^2 \quad -4y^2$
$0 = 10y - 4y^2$
$0 = 2y(5 - 2y)$

$5 \cdot 2 \cdot y$
$2 \cdot y \cdot y$
GCF = 2y

$\frac{2y}{2} = \frac{0}{2}$
$y=0$

$5-2y=0$
$-5 \quad -5$
$\frac{-2y}{-2} = \frac{-5}{-2}$
$y=\frac{5}{2}$

Example 14: Find the zeros of the function $f(x) = -2x^2 + x$.

$0 = -2x^2 + x$
$0 = x(-2x + 1)$

$-2 \cdot x \cdot x$
$1 \cdot x$
GCF = x

$x=0$

$-2x+1=0$
$-1 \quad -1$
$\frac{-2x}{-2} = \frac{-1}{-2}$
$x=\frac{1}{2}$

Vertical Motion Model

The height h (in feet) of a projectile can be modeled by

$h = -16t^2 + vt + s$

where t is the time (in seconds) the object has been in the air, v is the initial velocity (in feet per second), and s is the initial height (in feet).

Example 15: A startled armadillo jumps straight into the air with an initial velocity of 14 feet per second. After how many seconds does it land on the ground?

$s = 0$ (on the ground)

$h = 0$ (end on ground)

$0 = -16t^2 + 14t + 0$

$0 = -16t^2 + 14t$

$0 = 2t(-8t + 7)$

$\frac{2t}{2} = \frac{0}{2}$ $\quad t = 0$

$-8t + 7 = 0$

$-7 \quad -7$

$\frac{-8t}{-8} = \frac{-7}{-8}$

$t = \frac{7}{8}$ sec

Example 16: A fountain sprays water into the air with an initial velocity of 20 feet per second. $v = 20$ $\quad s = 0$

a) What is the height of the water after half a second?

$h = ?$ when $t = \frac{1}{2}$

$h = -16t^2 + 20t$

$h = -16\left(\frac{1}{2}\right)^2 + 20\left(\frac{1}{2}\right)$

$h = 6$ feet

b) When will the water land on the ground?

$t = ?$ when $h = 0$

$0 = -16t^2 + 20t$

$0 = 4t(-4t + 5)$

$\frac{4t}{4} = \frac{0}{4}$ $\quad t = 0$

$-4t + 5 = 0$

$-5 \quad -5$

$\frac{-4t}{-4} = \frac{-5}{-4}$

$t = \frac{5}{4}$ sec