

## 8.8 PART 1 Solve Radical Equations

A radical equation contains at least one radical expression with a variable in the radicand.

### A. Equation with One Radical ( $\sqrt{x}$ , $\sqrt{2x-1}$ , $\sqrt[3]{2x}$ )

Example 1:

$$3\sqrt{x+6} + 5 = 14$$

$$\frac{3\sqrt{x+6}}{3} = \frac{9}{3}$$

$$(\sqrt{x+6})^2 = (3)^2$$

$$x+6 = 9$$

$$x = 3$$

You must check your solutions! ✓

Steps:

1. Isolate the radical on one side of the =.
2. Raise both sides to the power of the index number on the radical.

$$3\sqrt{3+6} + 5 \stackrel{?}{=} 14$$

$$9 + 5 = 14 \checkmark$$

Example 2:

$$\sqrt{5x-1} + 8 = 2$$

$$(\sqrt{5x-1})^2 = (-6)^2$$

$$5x-1 = 36$$

$$\frac{5x}{5} = \frac{37}{5}$$

$$x = \frac{37}{5}$$

no solution

$$\sqrt{5 \cdot \frac{37}{5} - 1} + 8 \stackrel{?}{=} 2$$

$$6 + 8 \neq 2$$

Example 3:

$$\sqrt[3]{2x+1} + 2 = 4$$

$\begin{array}{cc} -2 & -2 \end{array}$

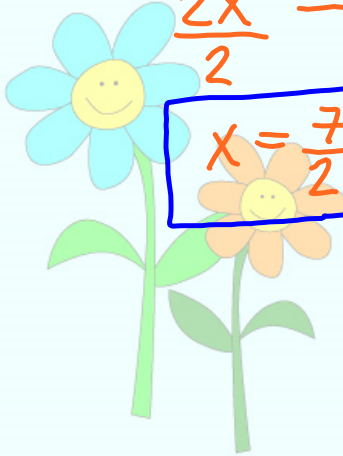
$$\left(\sqrt[3]{2x+1}\right)^3 = (2)^3$$

$$2x+1 = 8$$

$\begin{array}{cc} -1 & -1 \end{array}$

$$\frac{2x}{2} = \frac{7}{2}$$

$$x = \frac{7}{2}$$



$$\sqrt[3]{2 \cdot \frac{7}{2} + 1} + 2 \stackrel{?}{=} 4$$

$$2 + 2 = 4 \checkmark$$

Example 4:

$$\sqrt{2x-1} = (x-2)$$

$$2x-1 = (x-2)(x-2)$$

$$2x-1 = x^2 - 4x + 4$$

$\begin{array}{cc} -2x+1 & -2x+1 \end{array}$

$$0 = x^2 - 6x + 5$$

$$0 = (x-1)(x-5)$$

$$x \neq 1$$

$$x = 5$$



$$\sqrt{2 \cdot 1 - 1} \stackrel{?}{=} 1 - 2$$

$$1 \neq -1$$

$$\sqrt{2 \cdot 5 - 1} \stackrel{?}{=} 5 - 2$$

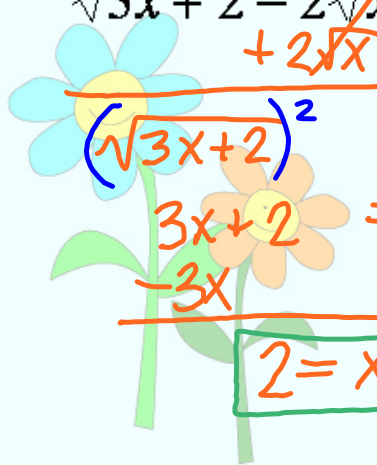
$$3 = 3 \checkmark$$

## B. Equation with Two Radicals

To solve:

1. Each side of the equation should have one radical.
2. You may need to move one radical by adding or subtracting it.

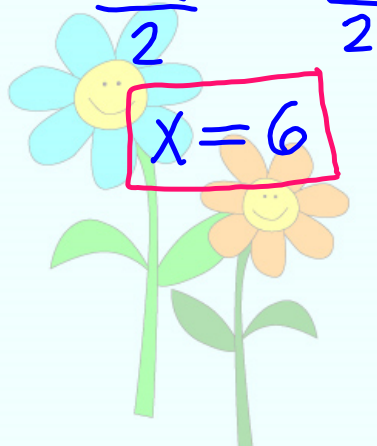
Example 1:



$$\begin{array}{r} \sqrt{3x+2} - 2\sqrt{x} = 0 \\ \quad \quad \quad + 2\sqrt{x} \quad \quad + 2\sqrt{x} \\ \hline (\sqrt{3x+2})^2 = (2\sqrt{x})^2 \\ 3x+2 = 4x \\ -3x \quad \quad -3x \\ \hline 2 = x \end{array}$$

$$\begin{array}{r} \sqrt{3 \cdot 2 + 2} - 2\sqrt{2} \stackrel{?}{=} 0 \\ \sqrt{8} - 2\sqrt{2} = 0 \\ 2\sqrt{2} - 2\sqrt{2} = 0 \end{array}$$

Example 2:



$$\begin{array}{r} (\sqrt[3]{x-5})^3 = (\sqrt[3]{7-x})^3 \\ x-5 = 7-x \\ \quad \quad \quad +5 \quad \quad +5 \\ \hline x = 12-x \\ +x \quad \quad +x \\ \hline 2x = 12 \\ \frac{2x}{2} = \frac{12}{2} \\ x = 6 \end{array}$$

$$\begin{array}{r} \sqrt[3]{6-5} \stackrel{?}{=} \sqrt[3]{7-6} \\ 1 = 1 \checkmark \end{array}$$

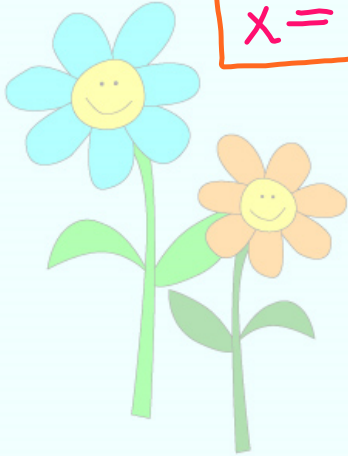
Example 3:

$$\left(\sqrt[4]{2x+1}\right)^4 = \left(\sqrt[4]{x+6}\right)^4$$

$$\begin{array}{r} 2x+1 = x+6 \\ -x \quad -x \\ \hline \end{array}$$

$$\begin{array}{r} x+1 = 6 \\ -1 \quad -1 \\ \hline \end{array}$$

$$x = 5$$



$$\begin{array}{l} \sqrt[4]{2 \cdot 5 + 1} \stackrel{?}{=} \sqrt[4]{5 + 6} \\ \sqrt[4]{11} = \sqrt[4]{11} \checkmark \end{array}$$

Example 4:

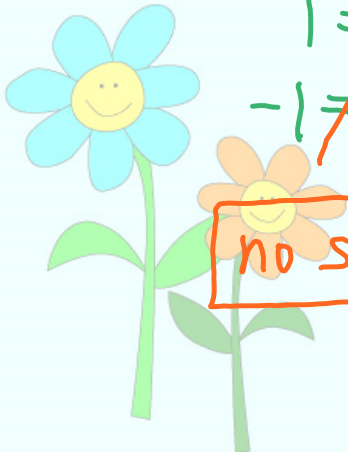
$$\begin{array}{l} \sqrt{2x+1} - \sqrt{x} = 0 \\ \quad \quad \quad +\sqrt{x} \quad +\sqrt{x} \\ \hline (\sqrt{2x+1})^2 = (\sqrt{x})^2 \end{array}$$

$$\begin{array}{r} 2x+1 = x \\ -2x \quad -2x \\ \hline \end{array}$$

$$1 = -x$$

$$-1 \neq x$$

no solution



$$\sqrt{2 \cdot 1 + 1} - \sqrt{-1} \stackrel{?}{\neq} 0$$

↑  
imag.

Example 5:

$$(\sqrt{x} - 1)^2 = (\sqrt{2x + 1})^2$$

$$(\sqrt{x} - 1)(\sqrt{x} - 1) = 2x + 1$$

$$\cancel{x} - \sqrt{x} - \sqrt{x} + \cancel{1} = 2x + 1$$

$$\frac{-2\sqrt{x}}{-2} = \frac{x}{-2}$$

$$(\sqrt{x})^2 = \left(-\frac{1}{2}x\right)^2$$

$$\cancel{-x} = \cancel{-x}^2$$

$$0 = \cancel{-x}^2 - \cancel{-x}$$

$$0 = x(\cancel{-x} - 1)$$

$$x=0: \sqrt{0} - 1 \stackrel{?}{=} \sqrt{2 \cdot 0 + 1}$$

$$0 - 1 = \sqrt{1}$$

$$-1 \neq 1$$

$$x=4$$

$$\sqrt{4} - 1 \stackrel{?}{=} \sqrt{2 \cdot 4 + 1}$$

$$2 - 1 = \sqrt{9}$$

$$1 \neq 3$$

no solution

$$x \neq 0$$

$$\frac{1}{4}x - 1 = 0$$

$$4 \cdot \frac{1}{4}x = 1 \cdot 4$$

$$x \neq 4$$

**HOMework****Solve.**

1.  $7 + \sqrt{4x + 8} = 9$

7.  $\frac{1}{6} \sqrt[3]{12x} = 1$

2.  $4\sqrt[3]{2x + 11} - 2 = 10$

8.  $\sqrt{6x - 4} = \sqrt{2x + 10}$

3.  $\sqrt[4]{x - 9} + 4 = 0$

9.  $\sqrt[3]{x + 2} = \sqrt[3]{x + 3}$

4.  $4\sqrt{3x} - 2 = 0$

10.  $x + \sqrt{x - 1} = 3$

5.  $2\sqrt{x} = 3\sqrt{x - 2}$

11.  $\sqrt{x + 10} = 2 - x$

6.  $\sqrt{3x + 4} = \sqrt{x} - 2$

12.  $2\sqrt{x - 2} = x - 2$