8.4 Greatest Common Factor & The Zero-Product Property

When two or more numbers are multiplied,
each number is a factor of the product.

Example 1: Name the factors of 24.

1, 2, 3, 4, 6, 8, 12, 24

Numbers that have only two factors, 1 and itself, are called prime numbers. They are whole numbers that are greater than 1.

Example 2: Name the first ten prime numbers.

2, 3, 5, 7, 11, 13, 17, 19, 23, 29

Whole numbers greater than 1 that are not prime are composite.

When a whole number is expressed as a product of factors that are all prime, the expression is called the prime factorization of the number.

Example: The prime factorization of 18 is 2 • 3 • 3.

$$\begin{array}{r|l} 2 & 18 \\ 3 & 9 \\ & 3 \end{array}$$

Example 3: Factor each monomial.

a) 200 $2^3 \cdot 5^2$

$2 \cdot 2 \cdot 2 \cdot 5 \cdot 5$

$$\begin{array}{r|l} 2 & 200 \\ 2 & 100 \\ 2 & 50 \\ 5 & 25 \\ & 5 \end{array}$$

b) 168 $2^3 \cdot 3 \cdot 7$

$2 \cdot 2 \cdot 2 \cdot 3 \cdot 7$

$$\begin{array}{r|l} 2 & 168 \\ 2 & 84 \\ 2 & 42 \\ 3 & 21 \\ & 7 \end{array}$$

c) $45x^2y^3$

$3 \cdot 3 \cdot 5 \cdot x \cdot x \cdot y \cdot y \cdot y$

$$\begin{array}{r|l} 5 & 45 \\ 3 & 9 \\ & 3 \end{array}$$

The greatest common factor (GCF) of two or more monomials is the product of their common factors.

Example 4: Find the GCF of the following.

a) 64 and 80

$\boxed{GCF = 16}$

2 | 64, 2 | 32, 2 | 16, 2 | 8, 2 | 4, 2

2 | 80, 2 | 40, 2 | 20, 2 | 10, 5

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 5$

$2 \cdot 2 \cdot 2 \cdot 2 = 16$

b) $40a^2b$ and $96ab^3$

$\boxed{GCF = 8ab}$

2 | 40, 2 | 20, 2 | 10, 5

2 | 96, 2 | 48, 2 | 24, 2 | 12, 2 | 6, 3

$2 \cdot 2 \cdot 2 \cdot 5 \cdot a \cdot a \cdot b$

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot a \cdot b \cdot b \cdot b$

$2 \cdot 2 \cdot 2 \cdot a \cdot b = 8ab$

c) 54, 72, 108

$\boxed{GCF = 18}$

2 | 54, 3 | 27, 3 | 9, 3

2 | 72, 2 | 36, 2 | 18, 3 | 9, 3

2 | 108, 2 | 54, 3 | 27, 3 | 9, 3

$2 \cdot 3 \cdot 3 \cdot 3$

$2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$

$2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$

$2 \cdot 3 \cdot 3 = 18$

Factoring Using the Distributive Property

Earlier in Chapter 8 we learned to use the distributive property to multiply a monomial and a polynomial.

Now we will work backwards to put polynomials in factored form.

Example 5: Use the distributive property to factor $10y^2 + 15y$.

$\underbrace{5y(2y + 3)}_{\text{factored form}}$

2 | 10, 5   3 | 15, 5

$2 \cdot 5 \cdot y \cdot y$

$3 \cdot 5 \cdot y$

GCF = 5y

Example 6: Use the distributive property to factor $18x^2 - 12x^3$.

$6x^2(3 - 2x)$

$3 \mid 18$, $2 \mid 6$, 3 $\quad$ $3 \mid 12$, $2 \mid 4$, 2

$2 \cdot 3 \cdot 3 \cdot x \cdot x$

$2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot x$

$GCF = 6x^2$

Example 7: Factor $16fg^2 - 24g^2h + 40g^2$.

$8g^2(2f - 3h + 5)$

$2 \cdot 2 \cdot 2 \cdot 2 \cdot f \cdot g \cdot g$

$2 \cdot 2 \cdot 2 \cdot 3 \cdot g \cdot g \cdot h$

$2 \cdot 2 \cdot 2 \cdot 5 \cdot g \cdot g$

$GCF = 8g^2$

Example 8: Factor $6k^3m + 14k^2m^2 - 4k^3m^3$.

$2k^2m(3k + 7m - 2km^2)$

$2 \cdot 3 \cdot k \cdot k \cdot k \cdot m$

$2 \cdot 7 \cdot k \cdot k \cdot m \cdot m$

$2 \cdot 2 \cdot k \cdot k \cdot k \cdot m \cdot m \cdot m$

$GCF = 2k^2m$

Solving Equations by Factoring

Zero Product Property

For all numbers a and b, if ab = 0, then a = 0, b = 0, or both a and b equal 0.

Example 9: Solve $\underbrace{(y + 2)(3y + 5)}_{\text{factored form}} = 0$. Then check each solution.

$y + \cancel{2} = 0$
$\quad -\cancel{2} \quad -2$
$\boxed{y = -2}$ ✓

$3y + 5 = 0$
$\quad -5 \quad -5$
$\frac{3y}{3} = \frac{-5}{3}$
$\boxed{y = -\frac{5}{3}}$ ✓

$y = -2$
$(-2+2)(3 \cdot -2 + 5) \stackrel{?}{=} 0$
$(0)(-1) = 0$ ✓

$y = -\frac{5}{3}$
$(-\frac{5}{3} + 2)(3 \cdot -\frac{5}{3} + 5) \stackrel{?}{=} 0$
$(\frac{1}{3})(0) = 0$ ✓

Example 10: Solve $(2a + 4)(a - 9) = 0$.

$2a + 4 = 0$
$\quad -4 \quad -4$
$\frac{2a}{2} = \frac{-4}{2}$
$\boxed{a = -2}$

$a - 9 = 0$
$\quad +9 \quad +9$
$\boxed{a = 9}$

Example 11: Solve $k^2 - 11k = 0$.

$k \cdot k$
$11 \cdot k$
$GCF = k$

factored form

$k(k - 11) = 0$

$\boxed{k = 0}$

$k - 11 = 0$
$+11 \quad +11$
$\boxed{k = 11}$

Example 12: Solve $-10n^2 = 35n$.

$+10n^2 \quad +10n^2$

$0 = 35n + 10n^2$

$0 = 5n(7 + 2n)$

$5 \cdot 7 \cdot n$
$5 \cdot 2 \cdot n \cdot n$
$GCF = 5n$

$\frac{5n}{5} = \frac{0}{5}$
$\boxed{n = 0}$

$7 + 2n = 0$
$-7 \quad -7$
$\frac{2n}{2} = \frac{-7}{2}$
$\boxed{n = -\frac{7}{2}}$

Example 13: Solve $4y^2 = 10y$.

$-4y^2 \quad -4y^2$

$0 = 10y - 4y^2$

$0 = 2y(5 - 2y)$

$5 \cdot 2 \cdot y$
$2 \cdot 2 \cdot y \cdot y$
$GCF = 2y$

$\frac{2y}{2} = \frac{0}{2}$
$\boxed{y = 0}$

$5 - 2y = 0$
$-5 \quad -5$
$\frac{-2y}{-2} = \frac{-5}{-2}$
$\boxed{y = 5/2}$

Example 14: Find the zeros of the function $f(x) = -2x^2 + x$.

$-2 \cdot x \cdot x$
$1 \cdot x$
$GCF = x$

$0 = -2x^2 + x$

$0 = x(-2x + 1)$

$\boxed{x = 0}$

$-2x + 1 = 0$
$-1 \quad -1$
$\frac{-2x}{-2} = \frac{-1}{-2}$
$\boxed{x = \frac{1}{2}}$

Vertical Motion Model

The height h (in feet) of a projectile can be modeled by

$$h = -16t^2 + vt + s$$

where t is the time (in seconds) the object has been in the air, v is the initial velocity (in feet per second), and s is the initial height (in feet).

Example 15: A startled armadillo jumps straight into the air with an initial velocity of 14 feet per second. After how many seconds does it land on the ground?

Example 16: A fountain sprays water into the air with an initial velocity of 20 feet per second.

a) What is the height of the water after half a second?

b) When will the water land on the ground?