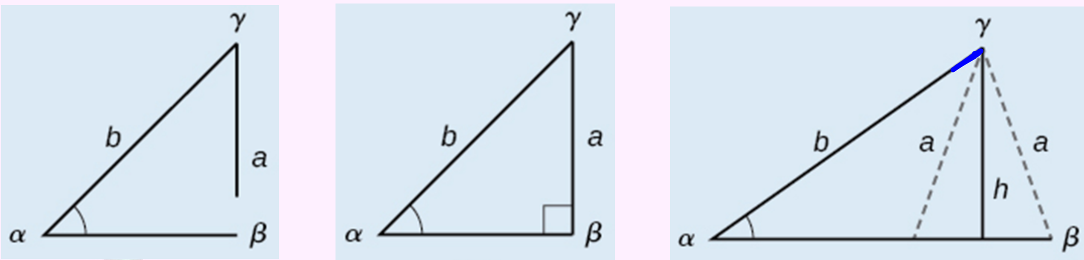
6.5 Part 2 The Law of Sines

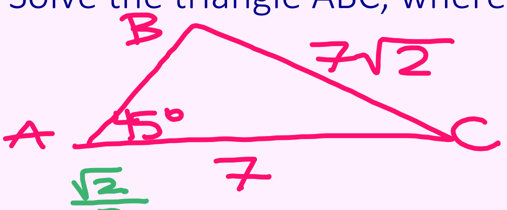
The Ambiguous Case

ASA and AAS give us one unique triangle.
However, if SSA is given, then there are three possibilities:
no triangles, 1 triangle, or 2 triangles.
Because of this, SSA is called the ambiguous case.



Example 1

Solve the triangle ABC, where A = 45°, a = 7√2, and b = 7.



$$\frac{\cancel{\sin 45°}^{\frac{\sqrt{2}}{2}}}{7\sqrt{2}} = \frac{\sin B}{7}$$

$$\frac{7\sqrt{2}\sin B}{7\sqrt{2}} = \frac{7\cancel{\sqrt{2}}}{2}\cdot\frac{1}{\cancel{7\sqrt{2}}}$$

$$\sin B = \frac{1}{2}$$

$$\boxed{m\angle B = 30°}$$

or ~~$m\angle B = 150°$~~

$$\boxed{m\angle C = 105°}$$

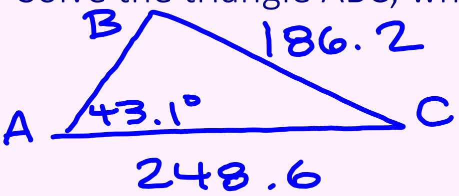
$$\frac{\sin 30°}{7} = \frac{\sin 105°}{c}$$

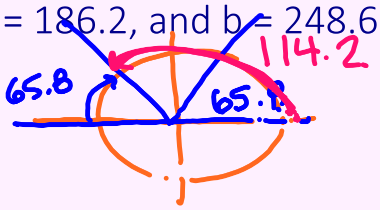
$$\frac{c\,\cancel{\sin 30°}}{\cancel{\sin 30°}} = \frac{7\sin 105°}{\sin 30°}$$

$$\boxed{c \approx 13.5}$$

Example 2

Solve the triangle ABC, where A = 43.1°, a = 186.2, and b = 248.6.





$$\frac{\sin 43.1^\circ}{186.2} = \frac{\sin B}{248.6}$$

$$\sin^{-1}\left(\frac{\cancel{186.2}(\sin B)}{\cancel{186.2}}\right) = \sin^{-1}\left(\frac{248.6 \sin 43.1}{186.2}\right)$$

$m\angle B \approx 65.8^\circ$ QI

or

$m\angle B \approx 114.2^\circ$ QII

$\triangle 1$

$m\angle B \approx 65.8^\circ$

$m\angle C \approx 71.1^\circ$

$$\frac{\sin 43.1}{186.2} = \frac{\sin 71.1}{c}$$

$c \approx 257.8$

$\triangle 2$

$m\angle B \approx 114.2^\circ$

$m\angle C \approx 22.7^\circ$

$$\frac{\sin 43.1}{186.2} = \frac{\sin 22.7}{c}$$

$c \approx$ [illegible]

Example 3

Solve the triangle ABC, where A = 42°, a = 70, and b = 122.

$$\frac{\sin 42^\circ}{70} = \frac{\sin B}{122}$$

$$\sin^{-1}\left(\frac{122 \sin 42^\circ}{70}\right) = \sin^{-1}(\sin B)$$

no $\triangle$