8.1 - 8.2 Part 3 Exponential Growth and Decay Word Problems

Exponential Growth Equation: $y = a(1 + r)^t$

a is the initial amount

r is the percent increase, written as a decimal

1 + r is the growth factor

EXAMPLE 1:

In 1995, there were $\overset{a}{275}$ cell phone subscribers in Aiken. The number of subscribers increased by 75% per year after 1995. Write an exponential equation that models the number of cell phone subscribers after t years. How many were there in 2004?

$r = 75\% = .75$

start 1995 end 2004 $t = 9$

$y = 275(1 + .75)^t$

$y = 275(1 + .75)^9$

$\boxed{y \approx 42{,}333 \text{ subscribers}}$

EXAMPLE 2:

In the exponential equation

$y = 35(1.27)^x$, identify:

a) the initial amount,

35

b) the growth factor, $1 + r$

1.27

c) the percent increase.

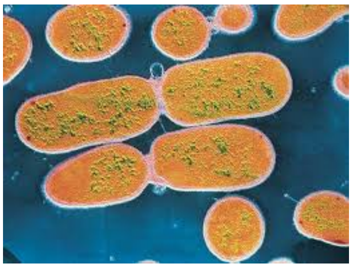
27%

$1.27 = 1 + r$

$-1 \quad -1$

$.27 = r$

EXAMPLE 3:



Bacteria can multiply at an alarming rate when each bacteria splits into two new cells, thus $\boxed{\text{doubling.}}$ If a scientist starts with three bacteria which can double every hour, how many bacteria will she have by the end of the day?

base of 2

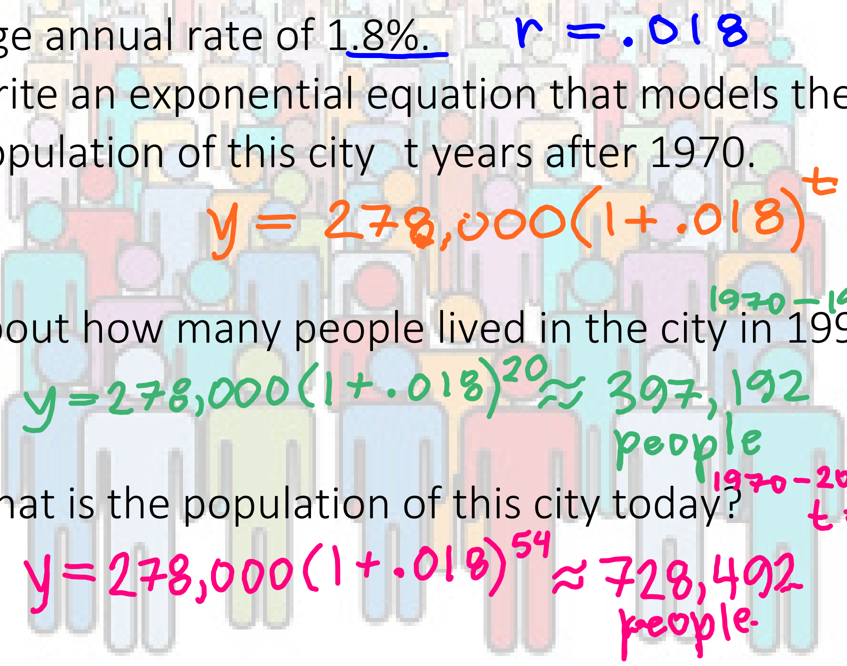
$a=3$

$t=24$

$y=3(2)^{24}$

$\boxed{y=50,331,648 \text{ bacteria}}$

EXAMPLE 4:



In 1970, the population of a city was about 278,000. Since then, the city population has grown at an average annual rate of 1.8%.

a

$r=.018$

a) Write an exponential equation that models the population of this city t years after 1970.

$y=278,000(1+.018)^t$

b) About how many people lived in the city in 1990?

$1970-1990 \rightarrow t=20$

$y=278,000(1+.018)^{20} \approx 397,192$ people

c) What is the population of this city today?

1970-2024

$t=54$

$y=278,000(1+.018)^{54} \approx 728,492$ people

Exponential Decay Equation: $y = a(1 - r)^t$

a is the initial amount

r is the percent decrease, written as a decimal

1 - r is the decay factor

EXAMPLE 5:

Jolene purchases a new car for $\overset{a}{\$22,499}$. The value of the car decreases by 11% each year. Write the exponential equation that models the car's value after t years. Then find its value after 3 years.

r = .11

$y = 22,499(1 - .11)^t$

$y = 22,499(1 - .11)^3$

$\boxed{y \approx \$15,861.10}$

EXAMPLE 6:

In the exponential equation

$y = 200(0.71)^x$, identify:

a) the initial amount,

200

b) the decay factor, $^{1-r}$

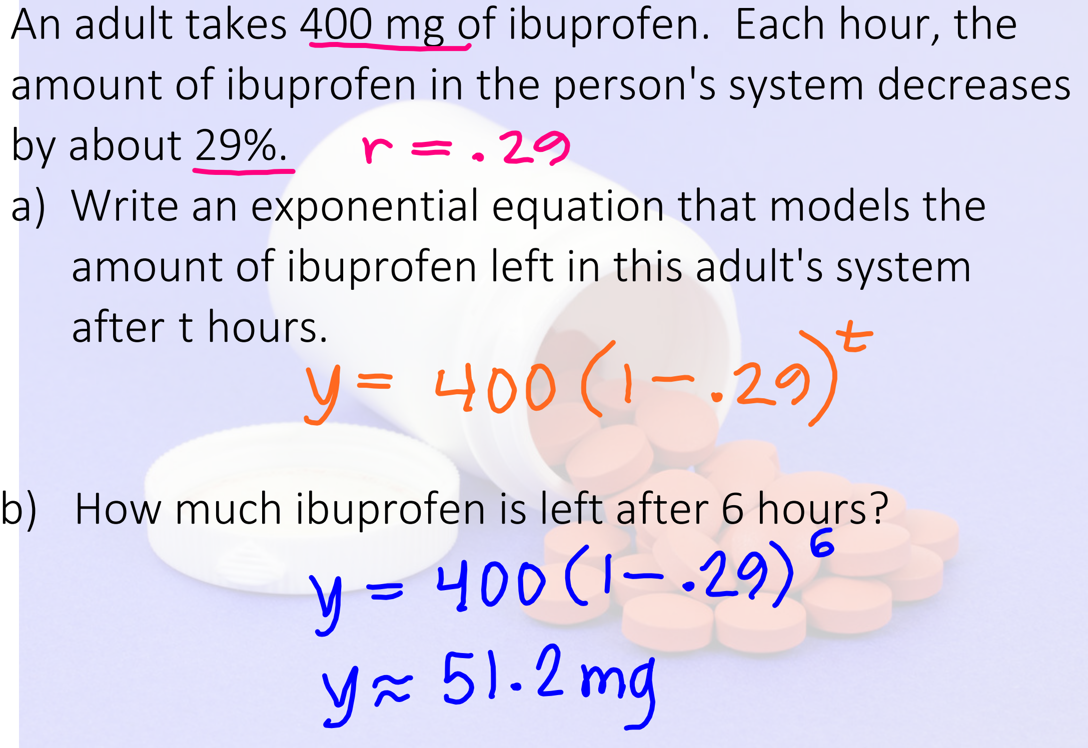
0.71

c) the percent decrease.

29%

$$\begin{aligned} 0.71 &= 1 - r \\ -1 &\quad\; -1 \\ \hline \frac{-0.29}{-1} &= \frac{-r}{-1} \\ 0.29 &= r \end{aligned}$$

EXAMPLE 7:



An adult takes $\underline{\overset{a}{400\text{ mg}}}$ of ibuprofen. Each hour, the amount of ibuprofen in the person's system decreases by about $\underline{29\%}$. $r = .29$

a) Write an exponential equation that models the amount of ibuprofen left in this adult's system after t hours.

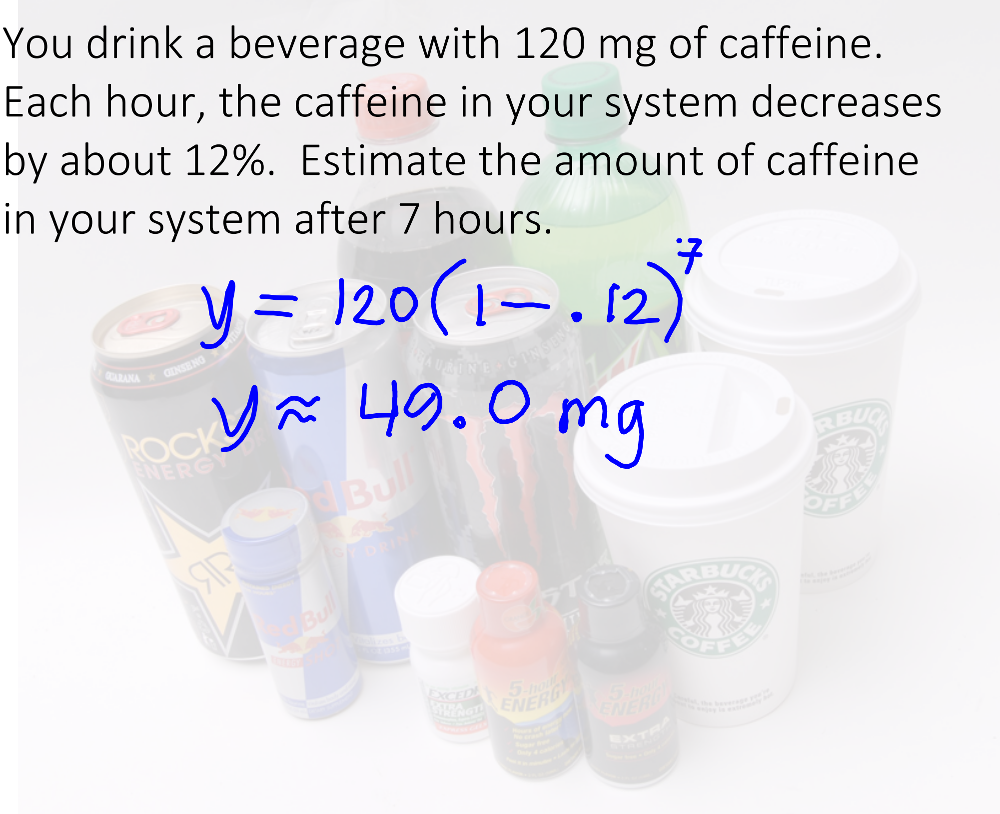
$$y = 400(1-.29)^t$$

b) How much ibuprofen is left after 6 hours?

$$y = 400(1-.29)^6$$

$$y \approx 51.2 \text{ mg}$$

EXAMPLE 8:



You drink a beverage with 120 mg of caffeine. Each hour, the caffeine in your system decreases by about 12%. Estimate the amount of caffeine in your system after 7 hours.

$$y = 120(1-.12)^7$$

$$y \approx 49.0 \text{ mg}$$