

8.7 SIMPLIFYING RADICAL EXPRESSIONS

Properties of Radicals

For any real number a ,

- $\sqrt[n]{a^n} = |a|$ if n is a positive even integer
- $\sqrt[n]{a^n} = a$ if n is a positive odd integer

EXAMPLES: Simplify each expression.

$$1. \sqrt[2]{49x^2y^5z^6}$$

$$\sqrt{\boxed{7 \cdot 7} \cdot \boxed{x \cdot x} \cdot \boxed{y \cdot y \cdot y} \cdot \boxed{z \cdot z} \cdot \boxed{z \cdot z} \cdot \boxed{z \cdot z}}$$

$$7|x|y^2|z^3|\sqrt{y}$$

$$2. \sqrt[3]{-27a^7b^3c^2}$$

$$\sqrt[3]{-27 \cdot \boxed{a \cdot a \cdot a} \cdot \boxed{a \cdot a \cdot a} \cdot \boxed{a} \cdot \boxed{b \cdot b \cdot b} \cdot c \cdot c}$$

$$-3a^2b\sqrt[3]{ac^2}$$

$$3. \sqrt[2]{64c^4de^6}$$

$$8c^2e^3\sqrt{d}$$

$$4. \sqrt[3]{-32f^6g^5h^2}$$

$$-2f^2g\sqrt[3]{4g^2h^2}$$

$$\begin{array}{r|l} 2 & 32 \\ -2 & 16 \\ -2 & 8 \\ -2 & 4 \\ -2 & 2 \end{array}$$

Product & Quotient Properties of Radicals

For $a \geq 0$, $b \geq 0$, and a positive integer n :

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

and

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} \quad (b \neq 0)$$

Simplify each expression. don't worry about abs. values

[Assume that the value of each variable is positive.]

5. $(27ab^3)^{\frac{1}{3}} \cdot \sqrt[3]{5a^4b}$

index

$$\sqrt[3]{27ab^3} \cdot \sqrt[3]{5a^4b}$$

$$\sqrt[3]{27 \cdot 5a^5b^4}$$

$$3ab \sqrt[3]{5a^2b}$$

6. $\frac{8(54x^5)^{\frac{1}{2}}}{4\sqrt{3x^3}}$

$$\frac{8 \cdot \sqrt{54x^5}}{4\sqrt{3x^3}}$$

$$\frac{2\sqrt{18x^2}}{2\sqrt{2 \cdot 9 \cdot x \cdot x}}$$

$$6x\sqrt{2}$$

$$\begin{aligned}
 7. \quad & \sqrt[3]{3r^2s^3} \cdot (9r^3s^4)^{\frac{1}{3}} \\
 & \sqrt[3]{3r^2s^3} \cdot \sqrt[3]{9r^3s^4} \\
 & \sqrt[3]{27r^5s^7} \\
 & 3rs^2 \sqrt[3]{r^2s}
 \end{aligned}$$

$$\begin{aligned}
 8. \quad & \frac{\sqrt{54x^3y^3}}{(3xy^2)^{\frac{1}{2}}} \cdot \frac{\sqrt{54x^3y^3}}{\sqrt{3xy^2}} \\
 & \sqrt{18x^2y} \\
 & 3x \sqrt{2y}
 \end{aligned}$$

$$\begin{aligned}
 9. \quad & (64ab^5)^{\frac{1}{2}} \cdot \sqrt{8ab^6} \\
 & \sqrt{64ab^5} \cdot \sqrt{8ab^6} \\
 & \sqrt{64 \cdot 8 a^2 b^{11}} \\
 & 16ab^5 \sqrt{2b}
 \end{aligned}$$

$$\begin{aligned}
 10. \quad & \frac{9\sqrt[3]{48x^8}}{(2x^3)^{\frac{1}{3}}} \\
 & \frac{9\sqrt[3]{48x^8}}{\sqrt[3]{2x^3}} \\
 & 9\sqrt[3]{24x^5} \\
 & 18x \sqrt[3]{3x^2}
 \end{aligned}$$

You may add or subtract radical expressions if the index and the radicand are the same.

Simplify each sum or difference.

$$\begin{aligned} 11. & (6 + \sqrt{12}) + (-7 + \sqrt{75}) \\ & \underline{6 + 2\sqrt{3}} \quad \underline{-7 + 5\sqrt{3}} \\ & -1 + 7\sqrt{3} \end{aligned}$$

$$\begin{aligned} 12. & (8 - \sqrt{45}) + (-2 + \sqrt{20}) \\ & \underline{8 - 3\sqrt{5}} \quad \underline{-2 - 2\sqrt{5}} \\ & 10 - 5\sqrt{5} \end{aligned}$$

Simplify each product.

$$13. \quad (-3 + 5\sqrt{2})(4 + 2\sqrt{2})$$

$$\begin{array}{ccccccc} (-3)(4) & + & (-3)(2\sqrt{2}) & + & (5\sqrt{2})(4) & + & (5\sqrt{2})(2\sqrt{2}) \\ \hline -12 & & -6\sqrt{2} & & +20\sqrt{2} & & -10\sqrt{2} + 20 \\ \hline & & 8 & + & 14\sqrt{2} & & \end{array}$$

$$14. \quad (-4\sqrt{6} + 1)(5 - 3\sqrt{6})$$

$$\begin{array}{ccccccc} (-4\sqrt{6})(5) & + & (-4\sqrt{6})(-3\sqrt{6}) & + & (1)(5) & + & (1)(-3\sqrt{6}) \\ \hline -20\sqrt{6} & & +72 & & +5 & & -3\sqrt{6} \\ \hline & & 77 & - & 23\sqrt{6} & & \end{array}$$

Rationalizing the denominator...

Write each expression with a rational denominator.

$$15. \quad \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\frac{\sqrt{3}}{3}$$

multiply by conjugate

$$16. \quad \frac{2}{(1 + \sqrt{3})} \cdot \frac{1 - \sqrt{3}}{(1 - \sqrt{3})}$$

$$\frac{2(1 - \sqrt{3})}{1 - \sqrt{3} + \sqrt{3} - 3}$$

$$\frac{2(1 - \sqrt{3})}{-2}$$

$$-1(1 - \sqrt{3})$$

$$-1 + \sqrt{3}$$