

8.6 PART 2 MORE ON RADICAL FUNCTIONS

Example 1

For each function, describe the transformations applied to $y = \sqrt{x}$.

$$a) y = -2\sqrt{x+3} - 4$$

vert. reflection
over x-axis

vert. stretch by
a factor of 2

hor. translation
3 units left

vert. translation
4 units down

$$b) y = \left(\frac{1}{2}\right)\sqrt{x-2} + 6$$

vert. compression
by a factor of $\frac{1}{2}$

hor. translation
2 units right

vert. translation
6 units up

Example 2

For each function, describe the transformations applied to $y = \sqrt{x}$.

$$a) y = \frac{3}{4}\sqrt{2x+6}$$

$$y = \left(\frac{3}{4}\right)\sqrt{2(x+3)}$$

vert. compression by
a factor of $\frac{3}{4}$

hor. compression by
a factor of $\frac{1}{2}$

hor. translation
left 3 units

$$b) y = -\frac{7}{2}\sqrt{3x-4} - 3$$

$$y = -\left(\frac{7}{2}\right)\sqrt{3\left(x-\frac{4}{3}\right)} - 3$$

vertical reflection
over x-axis

vert. stretch by a
factor of $\frac{7}{2}$

hor. comp. by a factor of $\frac{1}{3}$

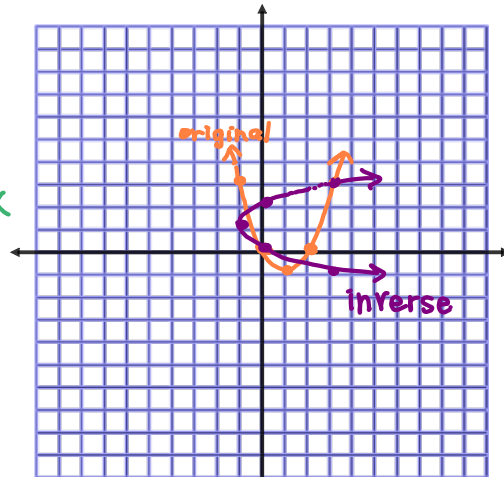
hor. translation $\frac{4}{3}$ units right

vert. translation 3 units down

Example 3 → switch x & y, solve for y

Find the inverse of $y = x^2 - 2x$. Then graph the function and its inverse together.

$$\begin{aligned} -x &= y^2 - 2y \\ \frac{-x}{-1} &= \frac{y^2 - 2y}{-1} \\ 0 &= y^2 - 2y - x \\ a=1 \quad b=-2 \quad c=-x \\ y &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ y &= \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-x)}}{2(1)} \\ y &= \frac{2 \pm \sqrt{4 + 4x}}{2} \\ y &= \frac{2 \pm \sqrt{4(1+x)}}{2} \\ y &= \frac{2 \pm 2\sqrt{1+x}}{2} \\ \boxed{y} &= 1 \pm \sqrt{1+x} \end{aligned}$$



$y = x^2 - 2x$
 $y = x(x - 2)$
 $x = 0 \quad x - 2 = 0$
 $x = 2$

x	y
-1	3
0	0
1	-1
2	0
3	3

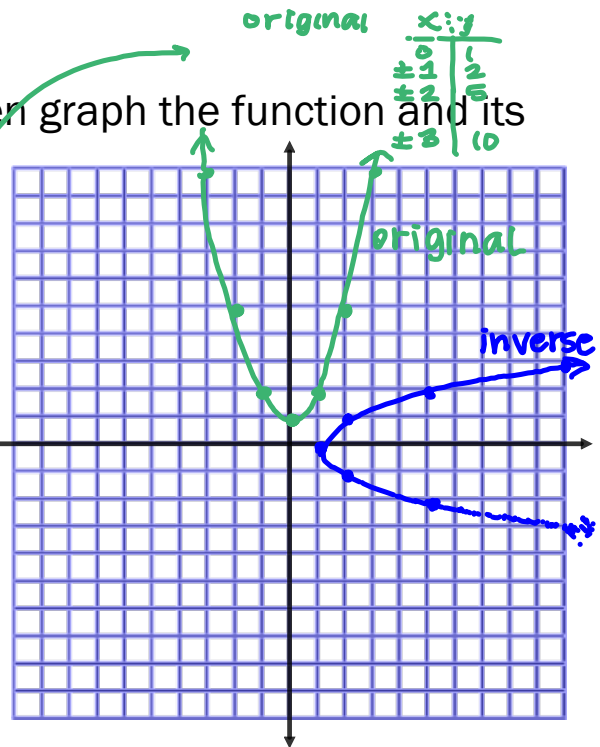
x	y
3	-1
0	0
-1	1
0	2
3	3

Example 4

Find the inverse of $y = x^2 + 1$. Then graph the function and its inverse together.

$$\begin{aligned} x &= y^2 + 1 \\ \frac{x-1}{-1} &= \frac{y^2 + 1}{-1} \\ \sqrt{x-1} &= \sqrt{y^2} \\ \pm \sqrt{x-1} &= y \\ x-1=0 &\rightarrow x=1 \\ x-1=1 &\rightarrow x=2 \\ x-1=4 &\rightarrow x=5 \\ x-1=9 &\rightarrow x=10 \end{aligned}$$

x	y
1	0
2	±1
5	±2
10	±3



x	y
0	1
±1	2
±2	5
±3	10

Example 5

Find the inverse of $y = x^2 + 3x - 4$. Then graph the function and its inverse together.

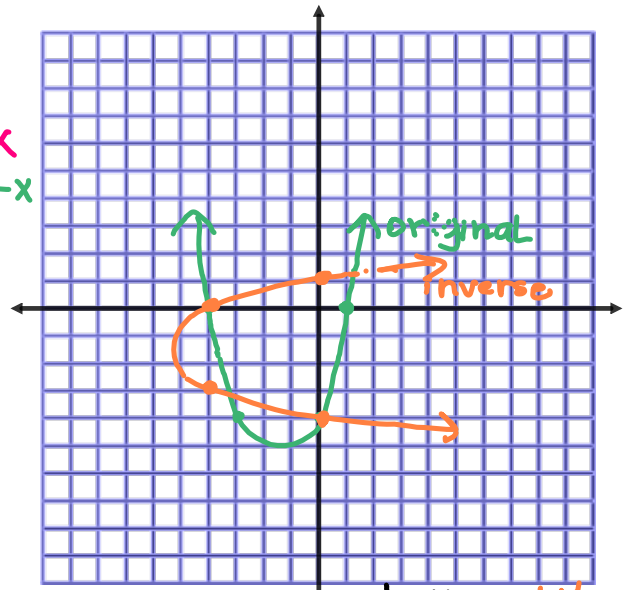
$$\begin{aligned} x &= y^2 + 3y - 4 \\ -x &= y^2 + 3y - 4 - x \\ 0 &= y^2 + 3y - 4 - x \\ a &= 1 \quad b = 3 \quad c = -4 - x \end{aligned}$$

$$y = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(-4-x)}}{2(1)}$$

$$y = \frac{-3 \pm \sqrt{9 + 16 + 4x}}{2}$$

$$y = \frac{-3 \pm \sqrt{25 + 4x}}{2}$$

inverse



$$\begin{aligned} y &= x^2 + 3x - 4 \\ y &= (x+4)(x-1) \\ x &= -4 \quad x = 1 \end{aligned}$$

x	y
1	0
-4	0
0	-4
0	1

x	y
0	1
1	0
-3	-4
-4	-3

Example 6

Evaluate each expression.

a) $3\sqrt[3]{27} - 5$

$$\begin{aligned} &3 \cdot 3 - 5 \\ &9 - 5 \\ &4 \end{aligned}$$

c) $-6\sqrt[3]{-8} - 2$

$$\begin{aligned} &-6 \cdot -2 - 2 \\ &12 - 2 \\ &10 \end{aligned}$$

b) $2(\sqrt[3]{-64})^2 + 7$

$$\begin{aligned} &2 \cdot (-4)^2 + 7 \\ &2(16) + 7 \\ &32 + 7 \\ &39 \end{aligned}$$

d) $4(\sqrt[3]{216})^2$

$$\begin{aligned} &4(6)^2 \\ &4(36) \\ &144 \end{aligned}$$