

Solving Right Triangles

Reminder:

S ine	C osine	T angent
O pposite	A djacent	O pposite
H ypotenuse	H ypotenuse	A djacent

So far we've learned to find *missing side measures*, but today we'll learn how to solve *missing angle measures*, too.

What if we get an equation like the ones below?
How will we solve them?

Example 1

$$\cancel{\sin^{-1}} (\cancel{\sin} x) = \frac{\cancel{\sin^{-1}}}{\cancel{\sin}} \left(\frac{5}{13} \right)$$

$$x \approx 22.6^\circ$$

Example 2

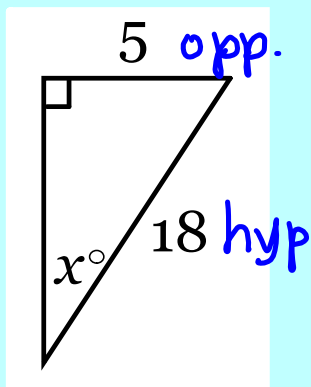
$$\cancel{\cos^{-1}} (\cancel{\cos} x) = \frac{\cancel{\cos^{-1}}}{\cancel{\cos}} \left(\frac{7}{10} \right)$$

$$x \approx 45.6^\circ$$

Example 3

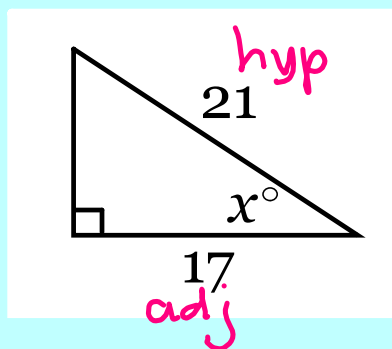
$$\cancel{\tan^{-1}} (\cancel{\tan} x) = \frac{\cancel{\tan^{-1}}}{\cancel{\tan}} \left(\frac{4}{21} \right)$$

$$x \approx 10.8^\circ$$

Example 4Solve for x .

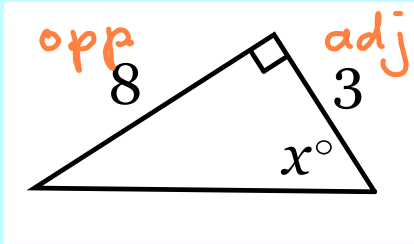
$$\cancel{\sin^{-1}} (\cancel{\sin x}) = \sin^{-1} \left(\frac{5}{18} \right)$$

$$x \approx 16.1^\circ$$

Example 5Solve for x .

$$\cancel{\cos^{-1}} (\cancel{\cos x}) = \cos^{-1} \left(\frac{17}{21} \right)$$

$$x \approx 36.0^\circ$$

Example 6Solve for x .

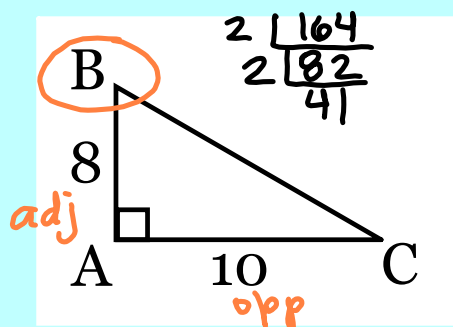
$$\cancel{\tan^{-1}} (\cancel{\tan x}) = \tan^{-1} \left(\frac{8}{3} \right)$$

$$x \approx 69.4^\circ$$

To *solve a right triangle* means to determine the measures of *all six parts* (3 angles, 3 sides).

Example 7

Solve the right triangle. Round decimals to the nearest tenth.



$$\begin{aligned} 8^2 + 10^2 &= c^2 \\ 64 + 100 &= c^2 \\ \sqrt{164} &= \sqrt{c^2} \\ 2\sqrt{41} &= c \end{aligned}$$

$$m\angle A = 90^\circ \quad AB = 8$$

$$m\angle B \approx 51.3^\circ \quad BC = 2\sqrt{41}$$

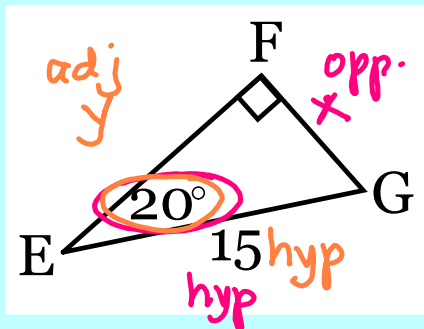
$$m\angle C \approx 38.7^\circ \quad CA = 10$$

$$\cancel{\tan^{-1}} (\cancel{\tan B}) = \tan^{-1} \left(\frac{10}{8} \right)$$

$$m\angle B \approx 51.3^\circ$$

Example 8

Solve the right triangle. Round decimals to the nearest tenth.



$$m\angle E = 20^\circ \quad EF \approx 14.1$$

$$m\angle F = 90^\circ \quad FG \approx 5.1$$

$$m\angle G = 70^\circ \quad GE = 15$$

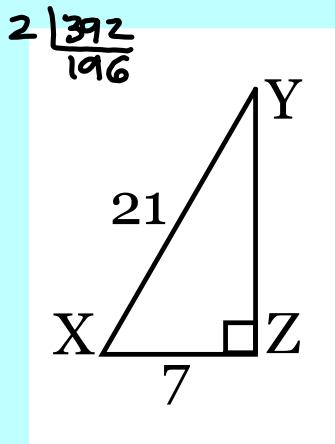
$$15 \cdot \sin 20^\circ = \frac{x}{15} \cdot 15 \quad 15 \cdot \cos 20^\circ = \frac{y}{15} \cdot 15$$

$$5.1 \approx x$$

$$14.1 \approx y$$

Example 9

Solve the right triangle. Round decimals to the nearest tenth.



$$m\angle X \approx 70.5^\circ \quad XY = 21$$

$$m\angle Y \approx 19.5^\circ \quad YZ = 14\sqrt{2}$$

$$m\angle Z = 90^\circ \quad ZX = 7$$

$$\cancel{\cos^{-1}}(\cos X) = \cos^{-1}\left(\frac{7}{21}\right)$$

$$m\angle X \approx 70.5^\circ$$

$$\begin{array}{r} 7^2 + b^2 = 21^2 \\ 49 + b^2 = 441 \\ -49 \quad -49 \\ \hline \sqrt{b^2} = \sqrt{392} \\ b = 14\sqrt{2} \end{array}$$