10.2 PART 2 ARCS & CHORDS

When a minor arc and a chord share the same endpoints, we call the arc the arc of the chord.

In the railroad crossing sign, \( \overparen{AB} \) is the arc of \( \overline{AB} \).

Theorem 10.4
In a circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \overparen{AE}, \overparen{AB} \cong \overparen{DC} )</td>
<td>( \circ ) given</td>
</tr>
<tr>
<td>2. ( \overparen{AE} \cong \overparen{CE}, \overparen{BE} \cong \overparen{DE} )</td>
<td>( \circ ) Radii in a circle are ( \cong )</td>
</tr>
<tr>
<td>3. ( \angle AEB \cong \angle CED )</td>
<td>( \circ ) Vert. ( \angle ) are ( \cong )</td>
</tr>
<tr>
<td>4. ( \triangle AEB \cong \triangle CED )</td>
<td>( \circ ) SAS</td>
</tr>
<tr>
<td>5. ( \overline{AB} \cong \overline{DC} )</td>
<td>( \circ ) CPCTC</td>
</tr>
</tbody>
</table>
A polygon is an **inscribed polygon** if each of its vertices lies on a circle.

Quadrilateral PQRS is inscribed in \( \odot C \).

**Example 1**
A stop sign is an octagon with congruent sides. It can be inscribed in a circle by using the center of the sign and a vertex of the sign as endpoints for the radius of the circle, as in QR. Find the measure of each of the right corresponding arcs of a circle around the stop sign shown below.

\[
\frac{360}{8} = 45
\]

\[45^\circ\]
Example 2
a) Find the measure of each minor arc created when an equilateral triangle is inscribed in a circle. \[\frac{360}{3} = 120^\circ\]

b) Find the measure of each minor arc created when a regular dodecagon is inscribed in a circle. \[\frac{360}{12} = 30^\circ\]

Theorem 10.5
In a circle, if a diameter is perpendicular to a chord, then it bisects the chord and its arc.

Theorem 10.6
If one chord is a perpendicular bisector of another chord, then the first chord is a diameter.
Example 3
Suppose a chord of a circle is 10 inches long and 12 inches from the center of the circle. Find the length of the radius.

\[
5^2 + 12^2 = r^2 \\
25 + 144 = r^2 \\
\sqrt{169} = r \\
13\text{in} = r
\]

Example 4
You discovered a crop circle in a nearby farm. A chord of the circle is 500 feet long and 600 feet from the center of the circle. Find the length of the radius.

\[
250^2 + 600^2 = r^2 \\
62500 + 360000 = r^2 \\
\sqrt{422500} = r \\
650\text{ft} = r
\]
Example 5
In \( \odot P \), \( AB = \overline{AC} \). Find the value of \( x \) as a radical.
\[
(3)^2 + (x + 2\sqrt{3})^2 = 6^2
\]
\[
9 + (x + 2\sqrt{3})^2 = 36
\]
\[
\sqrt{(x + 2\sqrt{3})^2} = 27
\]
\[
x + 2\sqrt{3} = 3\sqrt{3}
\]
\[
x = 1\sqrt{3}
\]

Theorem 10.7
In a circle or in congruent circles, two chords are congruent if and only if they are equidistant from the center.

Example 6
Chords \( \overline{CH} \) are \( \overline{IR} \) are equidistant from the center of \( \odot S \). If \( IR = 48 \), find \( CH \).
\[
x = 24
\]
\( CH = 48 \)