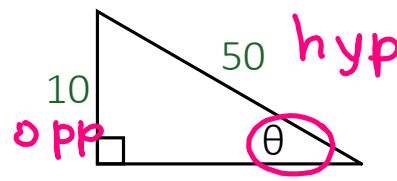## 6.4 Inverse Trigonometric Functions and Right Triangles

Example 1: Find the angle shown.

$$\sin^{-1}(\sin\theta) = \sin^{-1}\left(\frac{10}{50}\right)$$

$$\boxed{\theta \approx 11.5°}$$

opp — 10, hyp — 50, θ
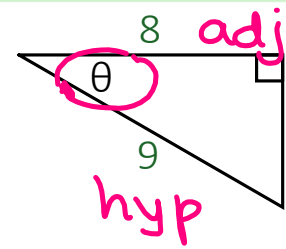
Example 2: Find the angle shown.

$$\cos^{-1}(\cos\theta) = \cos^{-1}\left(\frac{8}{9}\right)$$

$$\boxed{\theta \approx 27.3°}$$

adj — 8, hyp — 9, θ

Example 3: A 40-ft ladder leans against a building. If the base of the ladder is 6 ft from the base of the building, what is the angle formed by the ladder and the building?

hyp
40 ft

θ

6 ft
opp

$$\sin^{-1}(\sin \theta) = \sin^{-1}\left(\frac{6}{40}\right)$$
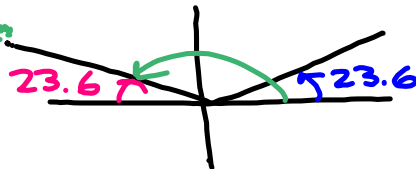
$$\theta \approx 8.6°$$
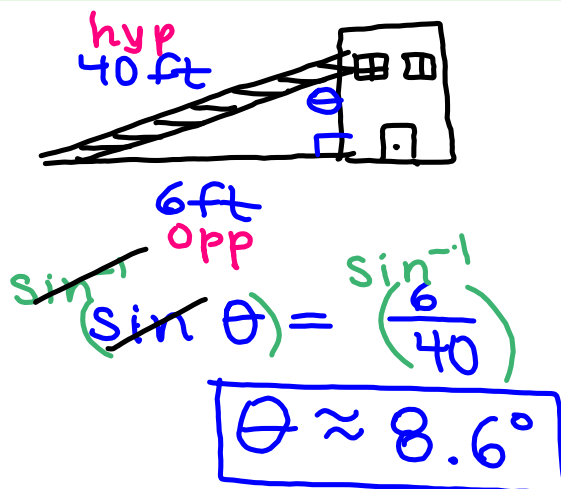
Example 4: A lighthouse is located on an island that is 2 miles off a straight shoreline. If the height of the lighthouse is 425 ft, find the angle formed by the beam of light and the shoreline.

Example 5: Find all angles between $0°$ and $180°$ satisfying the given equation.

$$\frac{S \mid A}{T \mid C}$$

$\sin^{-1}$ $\sin^{-1}$

a) $(\sin \theta = 0.4)$ QI, QII

$\theta \approx 23.6°, 156.4°$

23.6 $180-23.6$

$\cos^{-1}$ $\cos^{-1}$

b) $(\cos \theta = 0.4)$

$\theta \approx 66.4°$

Example 6: Find the exact value of the expression, if it is defined.
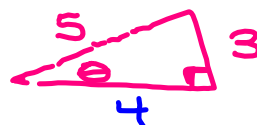
$$\sin^{-1} \frac{3}{5} = \theta$$

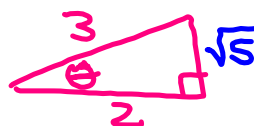a) $\cos(\sin^{-1} \frac{3}{5}) = \frac{4}{5}$

$\sin \theta = \frac{3}{5}$

$\cos \theta = \frac{4}{5}$

$3^2 + x^2 = 5^2$

$x = 4$

b) $\cos(\cos^{-1} \frac{2}{3}) = \frac{2}{3}$

$\cos \theta = \frac{2}{3}$

$\cos \theta = \frac{2}{3}$

$\sqrt{5}$ $2^2 + x^2 = 3^2$

$4 + x^2 = 9$

$x = \sqrt{5}$

Example 7: Find the exact value of the expression, if it is defined.

a) $\tan(\sin^{-1}\frac{4}{5}) = \frac{4}{3}$

$\sin\theta = \frac{4}{5}$

$\tan\theta = \frac{4}{3}$

$x^2 + 4^2 = 5^2$
$x = 3$
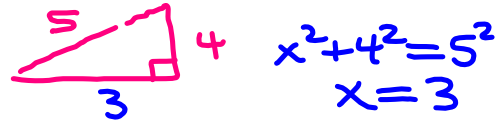
b) $\csc(\cos^{-1}\frac{7}{25}) = \frac{25}{24}$

$\cos\theta = \frac{7}{25}$

$\csc\theta = \frac{25}{24}$

$7^2 + x^2 = 25^2$
$x = 24$
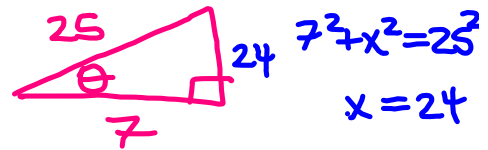
Example 8: Write $\sin(\cos^{-1}x)$ as an algebraic expression in x for $-1 \le x \le 1$.

$\cos\theta = \frac{x}{1}$

$\sin\theta = \frac{\sqrt{1-x^2}}{1}$

$= \sqrt{1-x^2}$

$x^2 + ?^2 = 1^2$
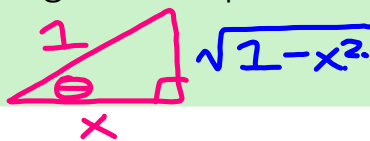$x^2 + ?^2 = 1$
$\sqrt{?^2} = \sqrt{1-x^2}$
$? = \sqrt{1-x^2}$

Example 9: Write $\tan(\cos^{-1}x)$ as an algebraic expression in x for $-1 \leq x \leq 1$.

$\cos\theta = x$

$\tan\theta = \dfrac{\sqrt{1-x^2}}{x}$