

## 8.5 Part 2 Solving Rational Inequalities

### RECALL: Solving Quadratic Inequalities

When **SOLVING** a quadratic inequality, you must solve as if set equal to zero and then do a **number line test** to find the solutions.

#### Example 1

Solve the inequality  $x^2 - 2x - 15 \geq 0$  and graph the solution on a number line.

$$\begin{array}{l}
 x-5=0 \rightarrow x=5 \\
 x+3=0 \rightarrow x=-3
 \end{array}
 \quad (x-5)(x+3) \geq 0$$

positive

$(-4-5)(-4+3)$      $(0-5)(0+3)$      $(6-5)(6+3)$   
 $(-9)(-1)$      $(-5)(3)$      $(1)(9)$   
 $9$      $-15$      $9$

$x \leq -3 \text{ or } x \geq 5$

### STEPS FOR SOLVING RATIONAL INEQUALITIES

1. Move all terms to one side (zero on the other).
2. Get a common denominator & condense into one fraction.
3. Set numerator and denominator equal to zero.
4. Make a number line to test the signs (positive/negative).
5. Solve. Be sure to check whether the inequality is satisfied by some or all of the endpoints of the intervals.

## Example 2

Solve the inequality  $\frac{x}{2x-1} \leq 1$ .

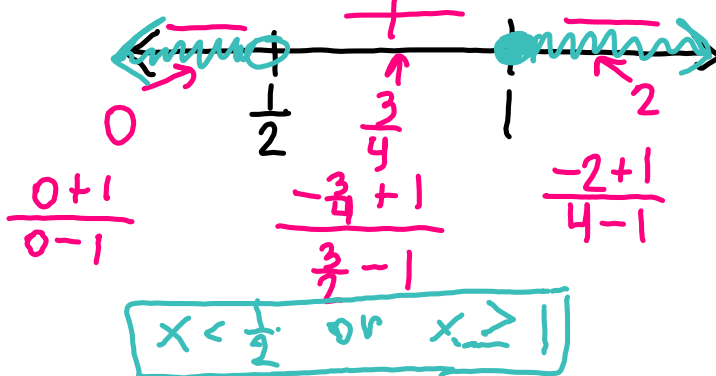
$$\begin{aligned} -x+1 &= 0 \\ x &= 1 \end{aligned}$$

$$\begin{aligned} 2x-1 &= 0 \\ x &= \frac{1}{2} \end{aligned}$$

$$\frac{x}{2x-1} - 1 \leq 0$$

$$\frac{x}{2x-1} + \frac{-2x+1}{2x-1} \leq 0$$

$$\frac{-x+1}{2x-1} \leq 0 \quad \text{neg. values}$$



$$x < \frac{1}{2} \text{ or } x > 1$$

## Example 3

Solve the inequality  $\frac{x-1}{x+2} > 1$ .

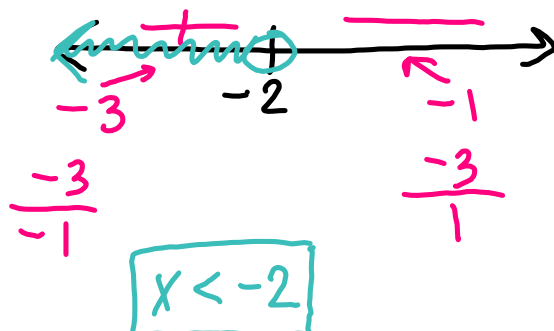
$$-3 \neq 0$$

$$\begin{aligned} x+2 &= 0 \\ x &= -2 \end{aligned}$$

$$\frac{x-1}{x+2} - 1 > 0$$

$$\frac{x-1}{x+2} + \frac{-x+2}{x+2} > 0$$

$$\frac{-3}{x+2} > 0 \quad \text{pos. values}$$



$$x < -2$$

## Example 4

Solve the inequality  $\frac{4x}{x-3} \leq 6$ .

$$\begin{aligned} -2x+18 &= 0 \\ x &= 9 \end{aligned}$$

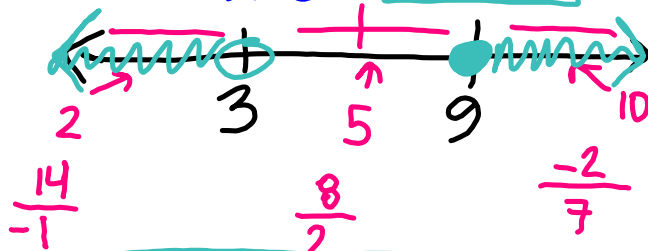
$$\begin{aligned} x-3 &= 0 \\ x &= 3 \end{aligned}$$

$$\frac{4x}{x-3} - 6 \leq 0$$

$$\frac{4x}{x-3} + \frac{-6(x-3)}{x-3} \leq 0$$

$$\frac{4x}{x-3} + \frac{-6x+18}{x-3} \leq 0$$

$$\frac{-2x+18}{x-3} \leq 0 \quad \text{neg. values}$$



$$x < 3 \text{ or } x \geq 9$$

## Example 5

Solve the inequality  $\frac{2}{x-1} > \frac{3}{x+4}$ .

$$\begin{aligned} -x+11 &= 0 \\ x &= 11 \end{aligned}$$

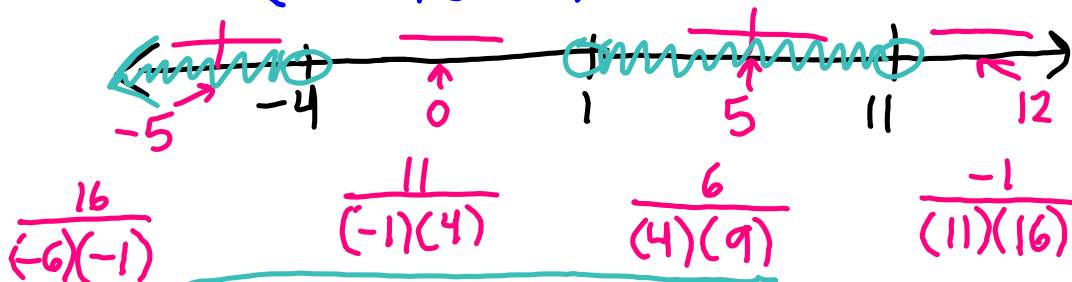
$$\begin{aligned} x-1 &= 0 \\ x &= 1 \end{aligned}$$

$$\begin{aligned} x+4 &= 0 \\ x &= -4 \end{aligned}$$

$$\frac{2(x+4)}{(x-1)(x+4)} - \frac{3(x-1)}{(x+4)(x-1)} > 0$$

$$\frac{2x+8}{(x-1)(x+4)} + \frac{-3x+3}{(x-1)(x+4)} > 0$$

$$\frac{-x+11}{(x-1)(x+4)} > 0 \quad \text{pos. values}$$



$$x < -4 \text{ or } 1 < x < 11$$

## Example 6

Solve the inequality  $\frac{x-2}{2(x-3)} \geq \frac{x}{x+3}$ .

$$\frac{(x-2)(x+3)}{2(x-3)(x+3)} - \frac{2x(x-3)}{2(x+3)(x-3)} \geq 0$$

$$\frac{x^2+x-6}{2(x-3)(x+3)} + \frac{-2x^2+6x}{2(x-3)(x+3)} \geq 0$$

$$\frac{-(x^2-7x+6)}{2(x-3)(x+3)} \geq 0$$

$$\frac{-(x-6)(x-1)}{2(x-3)(x+3)} \geq 0 \quad \text{pos. values}$$

$x=6 \quad x-6=0$   
 $x=1 \quad x-1=0$   
 $x=3 \quad x-3=0$   
 $x=-3 \quad x+3=0$

$\frac{-(-)(-)}{(-)(-)} \quad \frac{-(-)(-)}{(-)(+)} \quad \frac{-(-)(+)}{(-)(+)} \quad \frac{-(-)(+)}{(+)(+)} \quad \frac{-(+)(+)}{(+)(+)}$

$$\boxed{-3 < x \leq 1 \text{ or } 3 < x \leq 6}$$