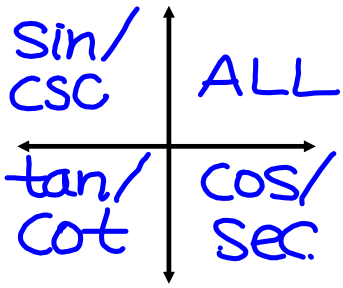
# 6.3 Trigonometric Functions of Angles

## Definition Review of the Trig Functions

| | |
|---|---|
| $\sin t = y$ | $\csc t = \frac{1}{y}$ |
| $\cos t = x$ | $\sec t = \frac{1}{x}$ |
| $\tan t = \frac{y}{x} \quad (x \neq 0)$ | $\cot t = \frac{x}{y} \quad (y \neq 0)$ |

If the terminal side falls on an axis, the angle is called a **quadrantal angle**.

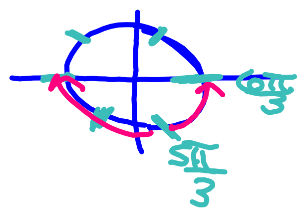
***Review of which trigonometric functions are positive in each quadrant:***




***Do you remember how to find the reference angle?***

**Example 1**

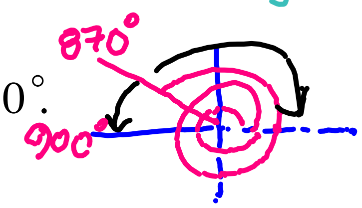
Find the reference angle for $\theta = \frac{5\pi}{3}$.




$\bar{t} = \frac{\pi}{3}$

**Example 2**

Find the reference angle for $\theta = 870^\circ$.

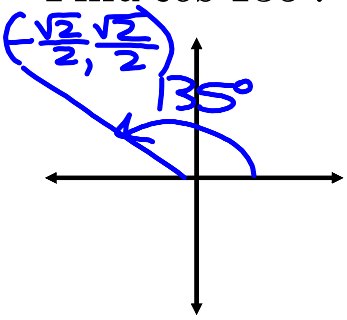



$\bar{t} = 30^\circ$

***More Review:*** ***To find the trigonometric function of an angle that is not acute, find its reference angle and use that value. Determine the sign of the answer based upon which quadrant the terminal side of the angle is in.***

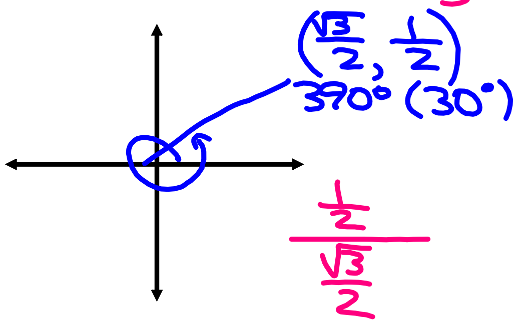
**Example 3**

Find cos 135°. $= -\frac{\sqrt{2}}{2}$



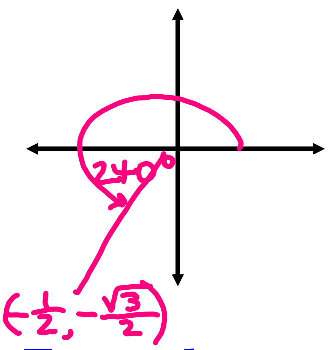

**Example 4**

Find tan 390°. $= \frac{\sqrt{3}}{3}$



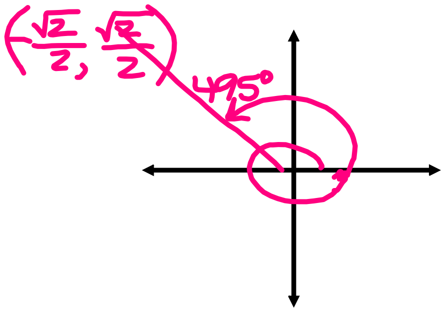

**Example 5**

Find sin 240°. $= -\frac{\sqrt{3}}{2}$




**Example 6**

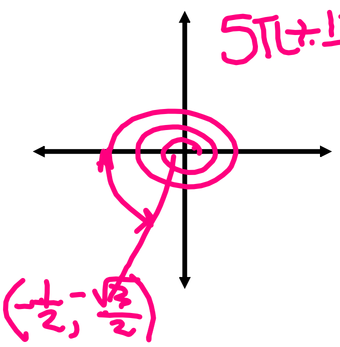
Find cot 495°. $= -1$




**Example 7**

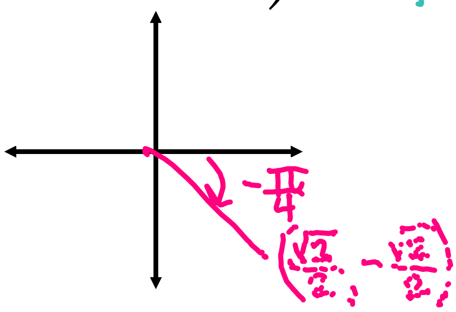
Find sin $\frac{16\pi}{3}$. $= -\frac{\sqrt{3}}{2}$

$5\pi + \frac{1\pi}{3}$




**Example 8**

Find cot $\left(\frac{-\pi}{4}\right)$. $= -1$



# *Review of Trigonometric Identities*

## Reciprocal Identities

$\csc\theta = \frac{1}{\sin\theta}$

$\tan\theta = \frac{1}{\cot\theta}$

$\sec\theta = \frac{1}{\cos\theta}$

$\cot\theta = \frac{1}{\tan\theta}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

## Pythagorean Identities

1. $\sin^2 t + \cos^2 t = 1$
2. $1 + \cot^2 t = \csc^2 t$
3. $\tan^2 t + 1 = \sec^2 t$

**Example 9**

in answer

Express sin θ in terms of cos θ.

$\sin\theta = \pm\sqrt{1-\cos^2\theta}$

$\sin^2\theta + \cancel{\cos^2\theta} = 1$
$\quad -\cancel{\cos^2\theta} \quad -\cos^2\theta$

$\sqrt{\sin^2\theta} = \sqrt{1-\cos^2\theta}$

**Example 10**

in answer

Express tan θ in terms of sin θ, where θ is in quadrant II.

$\tan\theta = \frac{\sin\theta}{-\sqrt{1-\sin^2\theta}}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ ✓

$\tan\theta = \frac{\sin\theta}{-\sqrt{1-\sin^2\theta}}$

$\cancel{\sin^2\theta} + \cos^2\theta = 1$
$-\cancel{\sin^2\theta} \quad -\sin^2\theta$

$\sqrt{\cos^2\theta} = \sqrt{1-\sin^2\theta}$

$\cos\theta = -\sqrt{1-\sin^2\theta}$

cos is neg. in QII

# Area of a Triangle

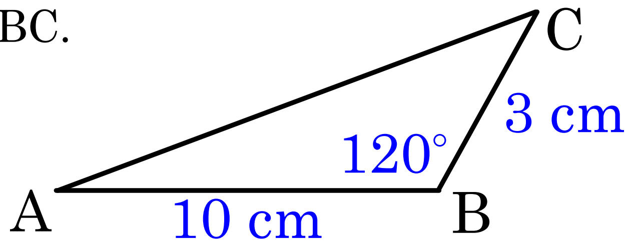
$$A = \frac{1}{2}ab\sin\theta$$

a and b are sides

$\theta$ is the included angle

**Example 11**

Find the area of Triangle ABC.



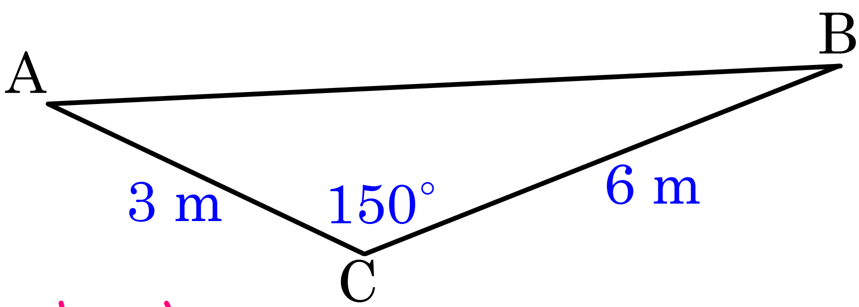

$A = \underbrace{\frac{1}{2}\cdot 10\cdot 3}\cdot \sin 120^\circ$

$A = 15 \cdot \frac{\sqrt{3}}{2}$

$\boxed{A = \frac{15\sqrt{3}}{2}\ cm^2}$

**Example 12**

Find the area of Triangle ABC.




$A = \frac{1}{2}ab\sin\theta$

$A = \underbrace{\frac{1}{2}\cdot 3\cdot 6}\cdot \sin 150^\circ$

$A = 9 \cdot \frac{1}{2}$

$\boxed{A = \frac{9}{2}m^2}$