6.6 – 6.7 Real Zeros of Polynomials

Rational Root Theorem

If a polynomial has integer coefficients, then every rational zero has the form:

#(no variable)

$$\pm\frac{p}{q} = \pm\frac{\text{factor of the constant}}{\text{factor of the leading coefficient}}$$

possible zeroes

I. List All Possible Rational Zeros

EXAMPLES:

leading coefficient

constant

1. $f(x) = 1x^3 + 2x^2 - 5x + 6$

p factors of constant term: ±1, ±2, ±3, ±6

q factors of leading coefficient: ±1

$\frac{p}{q}$ possible rational zeros: ±1, ±2, ±3, ±6

2. $f(x) = \overset{q}{2}x^3 - x^2 + 5x + \overset{p}{6}$

p: ±1, ±2, ±3, ±6

q: ±1, ±2

$\frac{p}{q}$: ±1, ±2, ±3, ±6, $\pm\frac{1}{2}$, $\pm\frac{3}{2}$

3. $f(x) = \overset{q}{6}x^4 + 35x^3 + 35x^2 - 55x - \overset{p}{21}$

p: ±1, ±3, ±7, ±21

q: ±1, ±2, ±3, ±6

$\frac{p}{q}$: ±1, ±3, ±7, ±21, $\pm\frac{1}{2}$, $\pm\frac{3}{2}$, $\pm\frac{7}{2}$, $\pm\frac{21}{2}$, $\pm\frac{1}{3}$, $\pm\frac{7}{3}$, $\pm\frac{1}{6}$, $\pm\frac{7}{6}$

Fundamental Theorem of Algebra

A polynomial of degree n has exactly n roots (zeros) in the set of complex numbers.

Roots or zeros may be rational (integers or fractions), irrational (square roots), or imaginary (i).

II. Find ALL Zeros

STEPS:

1. List all possible roots. $\frac{p}{q}$ list
2. Test each possibility until you find one zero.
3. Divide by the zero (using synthetic division) to get depressed polynomial.
4. Repeat steps 1 to 3 until the depressed polynomial is a quadratic.
5. Solve the quadratic by factoring, square roots, or the quadratic formula to get the last 2 zeros.

EXAMPLES: Find all the zeros.

4. $f(x) = 1x^3 - 5x^2 + 3x + 9$

p: ±1, ±3, ±9

q: ±1

$\boxed{\frac{p}{q}: \pm 1, \pm 3, \pm 9}$ list to use!

$$\cancel{\begin{array}{r|rrrr} -9 & 1 & -5 & 3 & 9 \\ & \downarrow & -9 & 126 & -1161 \\ \hline & 1 & -14 & 129 & \end{array}}$$

$$\begin{array}{r|rrrr} 3 & 1 & -5 & 3 & 9 \\ & \downarrow & 3 & -6 & -9 \\ \hline & 1 & -2 & -3 & 0 \end{array}$$

$(x-3)(x^2-2x-3)$

sum -2 prod -3

$\frac{-3}{1}$ $\frac{1}{1}$

$(x-3)(x-3)(x+1)$

$x-3=0$ $x-3=0$ $x+1=0$

$\boxed{x=3}$ $\boxed{x=-1}$

EXAMPLES: Find all the zeros.

5. $f(x) = 1x^3 + 3x^2 - 4$

$p: \pm 1, \pm 2, \pm 4$

$q: \pm 1$

$\boxed{\frac{p}{q}: \pm 1, \pm 2, \pm 4}$ list !

$$\begin{array}{r|rrrr} 1 & 1 & 3 & 0 & -4 \\ & \downarrow & 1 & 4 & 4 \\ \hline & 1 & 4 & 4 & 0 \end{array}$$

$(x-1)(x^2+4x+4)$

sum 4 product 4

$\frac{2}{1}$ $\frac{2}{1}$

$(x-1)(x+2)(x+2)$

$x-1=0 \quad x+2=0$

$\boxed{x=1,-2}$

EXAMPLES: Find all the zeros.

6. $1x^4 + 3x^3 - 12x^2 - 22x + 12 = 0$

$p: \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

$q: \pm 1$

$\boxed{\frac{p}{q}: \pm 1, \pm 2, \cancel{\pm 3}, \pm 4, \cancel{\pm 6}, \cancel{\pm 12}}$ list !

$$\begin{array}{r|rrrrr} 4 & 1 & 3 & -12 & -22 & 12 \\ & \downarrow & 4 & 28 & 64 & \\ \hline & 1 & 7 & 16 & 42 & \end{array}$$

$$\begin{array}{r|rrrrr} 2 & 1 & 3 & -12 & -22 & 12 \\ & \downarrow & 2 & 10 & -4 & -52 \\ \hline & 1 & 5 & -2 & -26 & \end{array}$$

$$\begin{array}{r|rrrrr} +3 & 1 & 3 & -12 & -22 & 12 \\ & \downarrow & 3 & 18 & 18 & -12 \\ \hline & 1 & 6 & 6 & -4 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -2 & 1 & 6 & 6 & -4 \\ & \downarrow & -2 & -8 & 4 \\ \hline & 1 & 4 & -2 & 0 \end{array}$$

$(x-3)(x^3+6x^2+6x-4)$

$(x-3)(x+2)(x^2+4x-2)$

$\boxed{x=3}$ $\boxed{x=-2}$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(-2)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{24}}{2}$$

$$\begin{array}{r|l} 2 & 24 \\ 2 & 12 \\ 2 & 6 \\ & 3 \end{array}$$

$$x = \frac{-4 \pm 2\sqrt{6}}{2}$$

$$\boxed{x = -2 \pm 1\sqrt{6}}$$