# 6.2 Part 2 Trigonometry of Right Triangles

## Review of Special Angles

$\sin\frac{\pi}{6} = \frac{1}{2}$ $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ $\tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$

$\csc\frac{\pi}{6} = 2$ $\sec\frac{\pi}{6} = \frac{2\sqrt{3}}{3}$ $\cot\frac{\pi}{6} = \sqrt{3}$

$\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ $\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ $\tan\frac{\pi}{4} = 1$

$\csc\frac{\pi}{4} = \sqrt{2}$ $\sec\frac{\pi}{4} = \sqrt{2}$ $\cot\frac{\pi}{4} = 1$

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ $\cos\frac{\pi}{3} = \frac{1}{2}$ $\tan\frac{\pi}{3} = \sqrt{3}$

$\csc\frac{\pi}{3} = \frac{2\sqrt{3}}{3}$ $\sec\frac{\pi}{3} = 2$ $\cot\frac{\pi}{3} = \frac{\sqrt{3}}{3}$

## Example 1

Evaluate without a calculator.

a) $\sin\frac{\pi}{6} + \cos\frac{\pi}{6}$

$\frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2}$

b) $\sin 30^\circ \csc 30^\circ$

$\frac{1}{2} \cdot 2 = 1$

c) $(\sin 60^\circ)^2 + (\cos 60^\circ)^2$

$\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{3}{4} + \frac{1}{4} = 1$

## Example 2

Solve the right triangle.

$m\angle A = 30°$

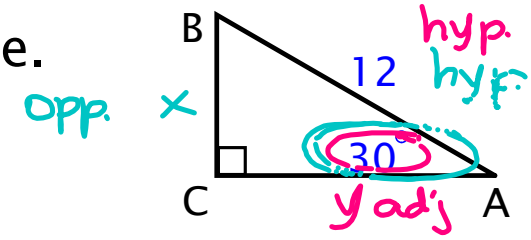
$m\angle B = 60°$

$m\angle C = 90°$

$AB = 12$

$BC = 6$

$AC = 6\sqrt{3}$




$12 \cdot \sin 30° = \frac{x}{12} \cdot 12$

$12 \cdot \sin 30° = x$

$12 \cdot \frac{1}{2} = x$

$6 = x$

$12 \cdot \cos 30° = \frac{y}{12} \cdot 12$

$6 \cdot \frac{\sqrt{3}}{2} = y$

$6\sqrt{3} = y$

## Example 3

Solve the right triangle.

$m\angle R \approx 32.2°$

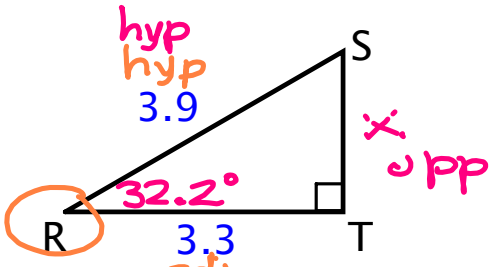
$m\angle S \approx 57.8°$

$m\angle T = 90°$

$RS = 3.9$

$ST \approx 2.1$

$TR = 3.3$




$\cos^{-1}(\cos R) = \cos^{-1}\left(\frac{3.3}{3.9}\right)$ (cos: adj)

$m\angle R \approx 32.2°$

$3.9 \cdot \sin 32.2° = \frac{x}{3.9} \cdot 3.9$

$2.1 \approx x$

line of sight– the line from an observer's eye to the object he is looking at

If the object being observed is above the horizontal, then the angle between the line of sight and the horizontal is called the angle of elevation.

If the object is below the horizontal, then the angle between the line of sight and the horizontal is called the angle of depression.

If the line of sight follows a physical object, such as an inclined plane or a hillside, the term angle of elevation is used.

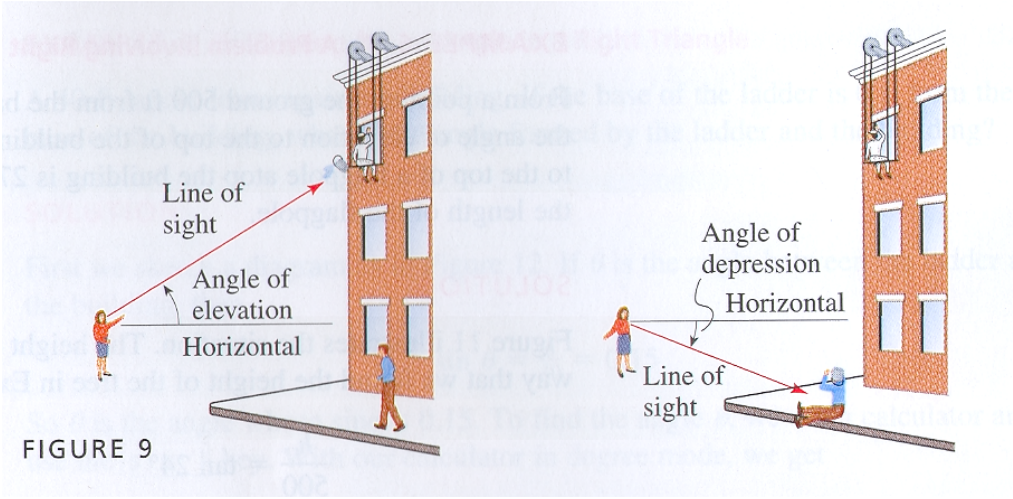


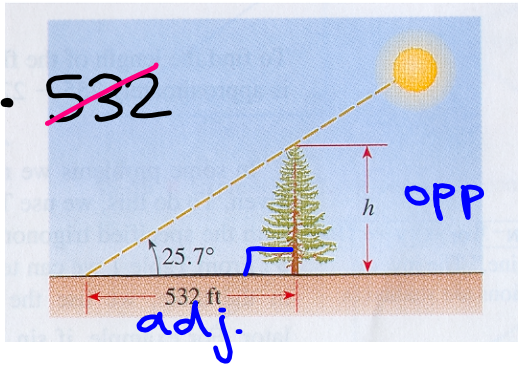
FIGURE 9

## Example 4

A giant redwood tree casts a shadow 532 feet long. Find the height of the tree if the angle of elevation of the sun is 25.7°.

$532 \cdot \tan 25.7° = \frac{h}{532} \cdot 532$

$h \approx 256.03 \text{ ft}$



## Example 5

From a point on the ground 500 feet from the base of a building, it is observed that the angle of elevation to the top of the building is 24° and the angle of elevation to the top of a flagpole atop the building is 27°. Find the height of the building and the length of the flagpole.

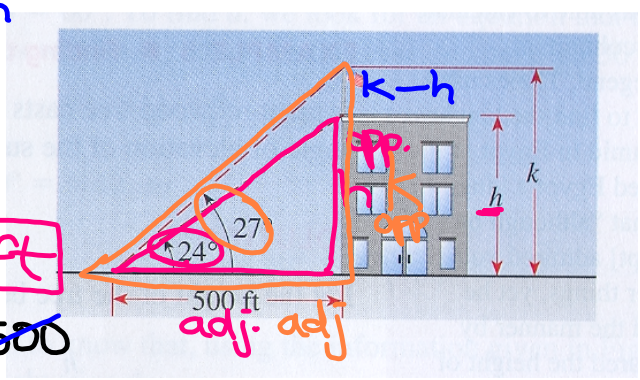
h k−h

500

$\tan 24° = \frac{h}{500} \cdot 500$

$h \approx 222.61$

building ≈ 222.61 ft



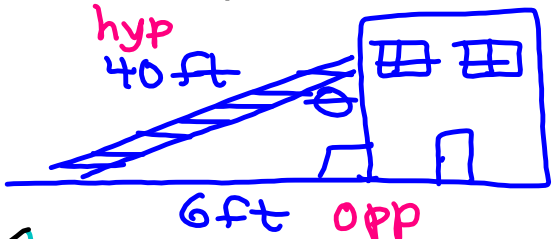

$500 \cdot \tan 27° = \frac{k}{500} \cdot 500$

$k \approx 254.76$

flagpole ≈ 32.15 ft

## Example 6

A 40-foot ladder leans against a building. If the base of the ladder is 6 feet from the base of the building, what is the angle formed by the ladder and the building?



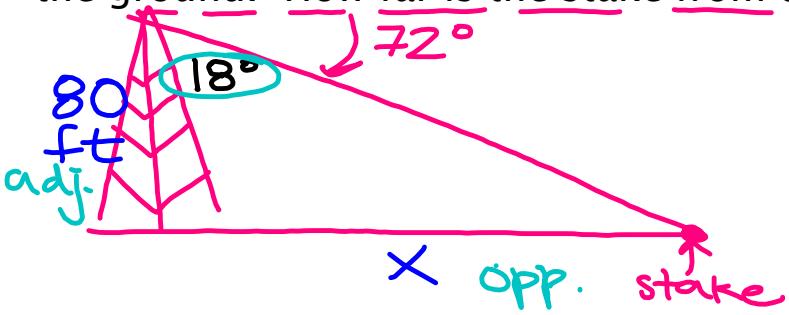

$\sin\theta = \frac{6}{40}$

$\theta = \sin^{-1}\left(\frac{6}{40}\right)$

$\theta \approx 8.63°$

## Example 7

From the top of a tower, the angle of depression to a stake on the ground is 72°. The top of the tower is 80 feet above the ground. How far is the stake from the foot of the tower?

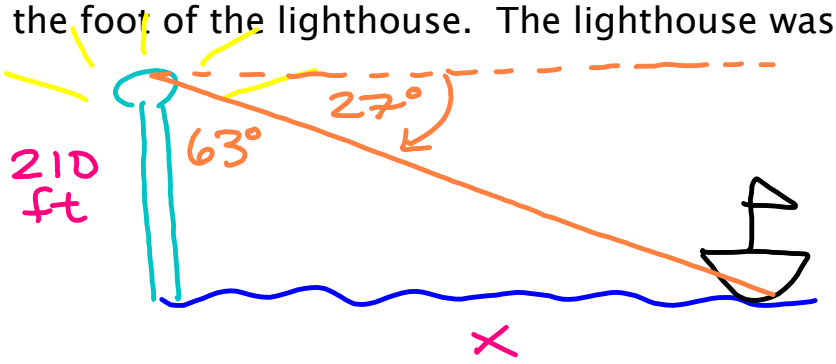



$$80 \cdot \tan 18^\circ = \frac{x}{80} \cdot 80$$

$$x \approx 26.0 \text{ ft}$$

## Example 8

From the top of a lighthouse 210 feet high, the angle of depression to a boat is 27°. Find the distance from the boat to the foot of the lighthouse. The lighthouse was built at sea level.




$$210 \cdot \tan 63^\circ = \frac{x}{210} \cdot 210$$

$$x \approx 412.15 \text{ ft}$$