



## 8.5 Solving Rational Equations

If each side of the equation is a single rational expression, you can cross multiply .

### Example 1

Solve.  $\frac{3}{x+1} = \frac{9}{4x+5}$

$$\begin{aligned} 9(x+1) &= 3(4x+5) \\ 9x+9 &= 12x+15 \\ \underline{-9x} \quad \underline{-9x} & \\ 9 &= 3x+15 \\ \underline{-15} \quad \underline{-15} & \\ -6 &= 3x \\ \frac{-6}{3} &= \frac{3x}{3} \quad \boxed{x=-2} \end{aligned}$$

When solving rational equations, always check for extraneous solutions !



### Example 2

Solve.  $\frac{1}{x-2} = \frac{x}{3}$

$$\begin{aligned} x(x-2) &= 3 \\ x^2-2x &= 3 \\ \underline{-3} \quad \underline{-3} & \\ x^2-2x-3 &= 0 \\ (x-3)(x+1) &= 0 \\ x-3=0 & \quad x+1=0 \\ \boxed{x=3} & \quad \boxed{x=-1} \end{aligned}$$

Example 3

Solve.  $\frac{1}{2x+5} = \frac{x}{11x+8}$

$$x(2x+5) = 1(11x+8)$$

$$\begin{array}{r} 2x^2 + 5x = 11x + 8 \\ -11x - 8 \quad -11x \quad -8 \\ \hline \end{array}$$

$$2x^2 - 6x - 8 = 0$$

$$2(x^2 - 3x - 4) = 0$$

$$2(x-4)(x+1) = 0$$

$$x-4=0 \quad x+1=0$$

$$\boxed{x=4}$$

$$\boxed{x=-1}$$

Example 4

Solve.  $\frac{2}{x-4} = \frac{x}{x+3}$

$$x(x-4) = 2(x+3)$$

$$\begin{array}{r} x^2 - 4x = 2x + 6 \\ -2x - 6 \quad -2x \quad -6 \\ \hline \end{array}$$

$$x^2 - 6x - 6 = 0$$

$$a=1 \quad b=-6 \quad c=-6$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(-6)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{60}}{2}$$

$$x = \frac{6 \pm 2\sqrt{15}}{2}$$

$$x = \frac{6}{2} \pm \frac{2\sqrt{15}}{2}$$

$$\boxed{x = 3 \pm \sqrt{15}}$$

$$\begin{array}{r} 2 \overline{)60} \\ 2 \overline{)30} \\ 3 \overline{)15} \\ \quad 5 \end{array}$$

When a rational equation is not expressed as a proportion, you can solve it by multiplying each side of the equation by the **LCD**

Example 5

LCD = 4x

Solve.  $4x \left( \frac{5}{x} + \frac{7}{4} \right) = \left( -\frac{9}{x} \right) 4x$

$$\frac{4x \cdot 5}{\cancel{x}} + \frac{4x \cdot 7}{4} = -\frac{9}{\cancel{x}} \cdot \frac{4x}{1}$$

$$\frac{20}{-20} + 7x = \frac{-36}{-20}$$

$$\frac{7x}{7} = \frac{-56}{7}$$

$$x = -8$$

Example 6

LCD = 2(x-2)

Solve.  $\frac{3}{2} + \frac{2}{\frac{2x-4}{2(x-2)}} = \frac{1}{x-2} \leftarrow 2-2 \neq 0$

$$\frac{2(x-2) \cdot 3}{2} + \frac{2(x-2) \cdot 2}{2(x-2)} = \frac{2(x-2) \cdot 1}{x-2}$$

$$3(x-2) + 2 = 2$$

$$3x - 6 + 2 = 2$$

$$3x - 4 = 2$$

$$3x = 6$$

$$x \neq 2$$

no solution

$$\text{LCD} = x(x-5)$$

### Example 7

Solve.  $1 - \frac{8}{x-5} = \frac{3}{x}$

$$x(x-5) \cdot 1 - \frac{x(x-5) \cdot 8}{\cancel{x-5}} = \frac{x(x-5) \cdot 3}{\cancel{x}}$$

$$\begin{aligned} x(x-5) - 8x &= 3(x-5) \\ x^2 - 5x - 8x &= 3x - 15 \\ -3x + 15 & \end{aligned}$$

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$$x^2 - 16x + 15 = 0$$

$$(x-15)(x-1) = 0$$

$$\boxed{x=15 \quad x=1}$$

$$\text{LCD} = (x-3)(x+3)$$

### Example 8

Solve.  $\frac{6}{x-3} = \frac{8x^2}{x^2-9} - \frac{4x}{x+3}$

$$\frac{(x-3)(x+3) \cdot 6}{\cancel{x-3}} = \frac{(x-3)(x+3) \cdot 8x^2}{\cancel{(x-3)(x+3)}} - \frac{(x-3)(x+3) \cdot 4x}{\cancel{x+3}}$$

$$6(x+3) = 8x^2 - 4x(x-3)$$

$$6x + 18 = 8x^2 - 4x^2 + 12x$$

$$6x + 18 = 4x^2 + 12x$$

$$\begin{array}{r} -6x \quad -18 \\ \hline 0 = 4x^2 + 6x - 18 \end{array} \quad \begin{array}{r} 53 \text{ p. } 18 \\ \frac{3}{1} \cdot \frac{6}{2} = \frac{3}{2} \end{array}$$

$$0 = 4x^2 + 6x - 18$$

$$0 = 2(2x^2 + 3x - 9)$$

$$0 = 2(x+3)(2x-3)$$

$$\cancel{x=-3} \quad \boxed{x = \frac{3}{2}}$$

$$\text{LCD} = (x+3)(x+2)$$

### Example 9

Solve.  $\frac{3x}{x+2} - \frac{2}{x+3} = \frac{36}{x^2+5x+6}$

$$\cancel{(x+3)}\cancel{(x+2)} \cdot \frac{3x}{x+2} - \cancel{(x+3)}\cancel{(x+2)} \cdot \frac{2}{x+3} = \cancel{(x+3)}\cancel{(x+2)} \cdot \frac{36}{\cancel{(x+3)}\cancel{(x+2)}}$$

$$3x(x+3) - 2(x+2) = 36$$

$$3x^2 + 9x - 2x - 4 = 36$$

$$3x^2 + 7x - 40 = 0$$

$$(x+5)(3x-8) = 0$$

$$x = -5 \quad x = \frac{8}{3}$$

$$\begin{array}{r} 57p-120 \\ 5 \quad 17 \quad -9 \\ 1 \quad 3 \quad 3 \end{array}$$

$$\text{LCD} = (x-3)(x+3)$$

### Example 10

Solve.  $1 + \frac{3}{x^2-9} = \frac{10}{x+3}$

$$\cancel{(x-3)}\cancel{(x+3)} \cdot 1 + \cancel{(x-3)}\cancel{(x+3)} \cdot \frac{3}{\cancel{(x-3)}\cancel{(x+3)}} = \cancel{(x-3)}\cancel{(x+3)} \cdot \frac{10}{\cancel{(x-3)}\cancel{(x+3)}}$$

$$(x-3)(x+3) + 3 = 10(x-3)$$

$$x^2 - 9 + 3 = 10x - 30$$

$$-10x + 30 = -10x + 30$$

$$x^2 - 10x + 24 = 0$$

$$(x-6)(x-4) = 0$$

$$x = 6 \quad x = 4$$