

## 7.3 Similar Right Triangles

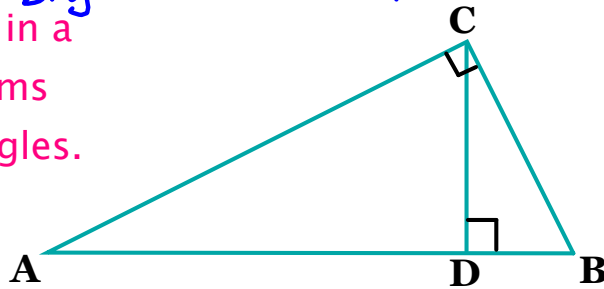
### Theorem 7.5

If the altitude is drawn from to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.

$$\triangle ABC \sim \triangle ACD \sim \triangle CBD$$

big
medium
little

An altitude drawn in a right triangle forms three similar triangles.



### Theorem 7.6

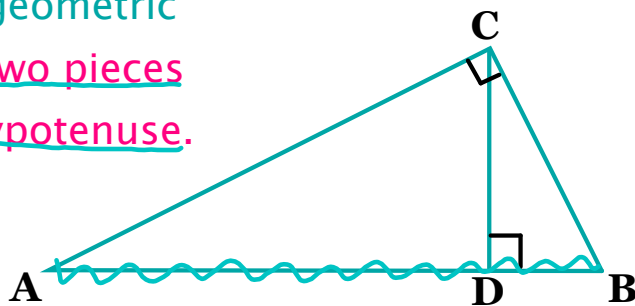
In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of the altitude is the geometric mean of the lengths of the two segments.

CD  
The altitude is the geometric mean between the two pieces that make up the hypotenuse.

AD & DB

$$\frac{AD}{CD} = \frac{CD}{DB}$$



**Example 1**

To find the height of his house, Kevin held a book near his eye so that the top and bottom of the house were in line with the edges of the cover. If his eye level is 5 feet off the ground and he is standing about 10 feet from the house, how tall is the house?

height = 25 ft

altitude 10 ft

5 ft

hypotenuse

$x = 20$

**Example 2**

Find the value of  $x$ .

a)

$x$  alt.

2 9

$\frac{2}{x} = \frac{x}{9}$

$\sqrt{x^2} = \sqrt{18}$

$2 \overline{)18}$   
 $3 \overline{)9}$   
 $3$

$x = 3\sqrt{2}$

b)

10

alt.  $x$

6

$\frac{10}{x} = \frac{x}{6}$

$\sqrt{x^2} = \sqrt{60}$

$2 \overline{)60}$   
 $2 \overline{)30}$   
 $3 \overline{)15}$   
 $5$

$x = 2\sqrt{15}$

**Theorem 7.7**

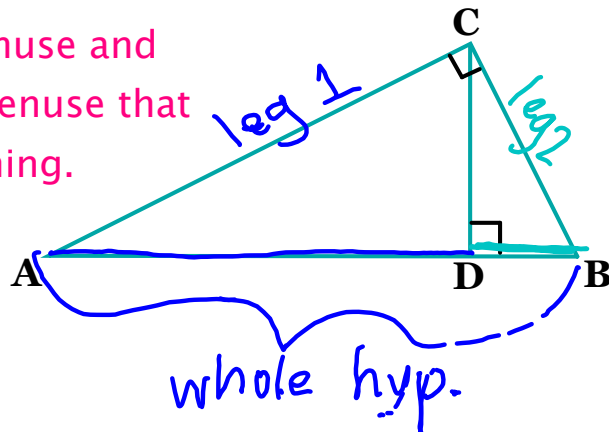
In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of each leg of the right triangle is the geometric mean of the lengths of the hypotenuse and the segment of the hypotenuse that is adjacent to the leg.

The leg is the geometric mean between the hypotenuse and the piece of the hypotenuse that the leg is touching.

$$\frac{AB}{AC} = \frac{AC}{AD}$$

$$\frac{AB}{CB} = \frac{CB}{DB}$$

**Example 3**

Find the values of  $x$  and  $y$ .

$$\frac{6}{x} = \frac{x}{2}$$

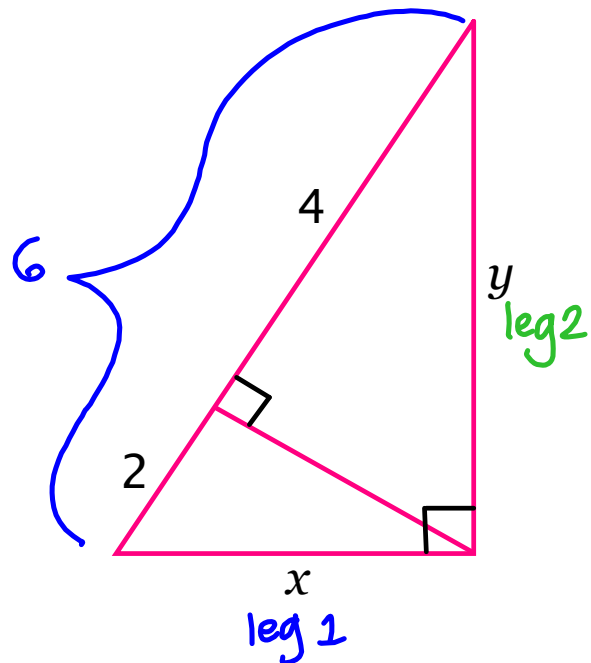
$$\sqrt{x^2} = \sqrt{12}$$

$$x = 2\sqrt{3}$$

$$\frac{6}{y} = \frac{y}{4}$$

$$\sqrt{y^2} = \sqrt{24}$$

$$y = 2\sqrt{6}$$



*Example 4*Find the values of  $x$  and  $y$ .

$$\frac{13}{x} = \frac{x}{8}$$

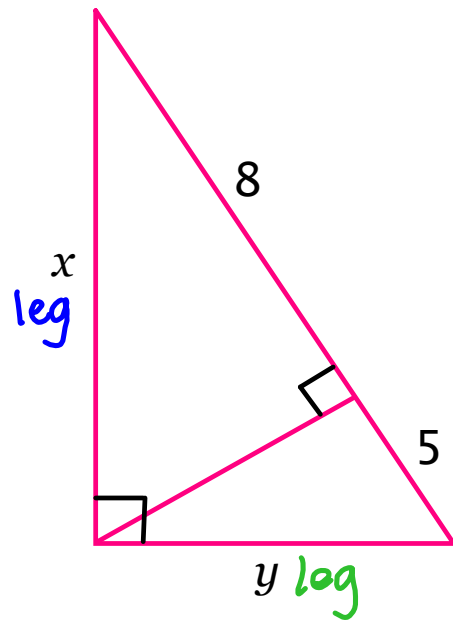
$$\sqrt{x^2} = \sqrt{104}$$

$$x = 2\sqrt{26}$$

$$\frac{13}{y} = \frac{y}{5}$$

$$\sqrt{y^2} = \sqrt{65}$$

$$y = \sqrt{65}$$

*Example 5*Find the values of  $x$ ,  $y$ , and  $z$ .

$$\frac{\text{leg}}{z} = \frac{z}{7} \quad \frac{\text{leg}}{y} = \frac{y}{5}$$

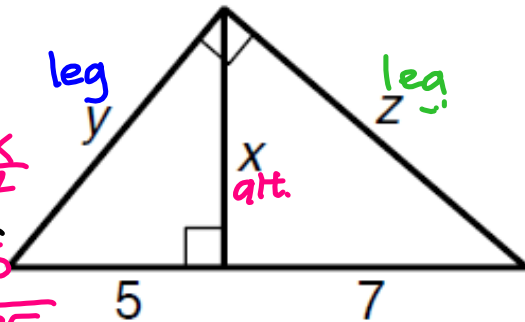
$$\sqrt{z^2} = \sqrt{84} \quad \sqrt{y^2} = \sqrt{60}$$

$$z = 2\sqrt{21} \quad y = 2\sqrt{15}$$

$$\frac{\text{alt}}{x} = \frac{x}{7}$$

$$\sqrt{x^2} = \sqrt{35}$$

$$x = \sqrt{35}$$

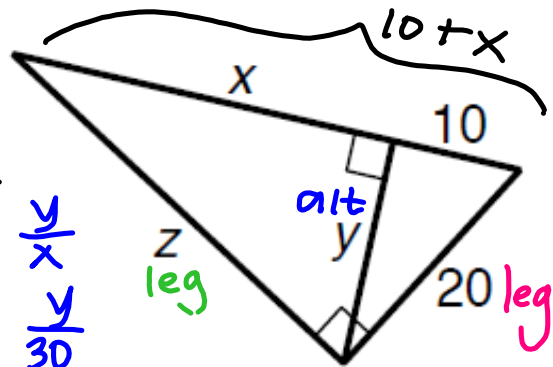


## Example 6

Find the values of  $x$ ,  $y$ , and  $z$ .

<u>leg</u>	<u>leg</u> <del>start</del>	<u>alt</u>
$\frac{10+x}{z} = \frac{z}{x}$	$\frac{10+x}{20} = \frac{20}{10}$	$\frac{10}{y} = \frac{y}{x}$
$\downarrow$	$10(10+x) = 400$	$\frac{10}{y} = \frac{y}{30}$
$\frac{40}{z} = \frac{z}{30}$	$100 + 10x = 400$	$\sqrt{y^2} = \sqrt{300}$
$\sqrt{z^2} = \sqrt{1200}$	$\frac{10x - 300}{10} = \frac{300}{10}$	$100 \cdot 3$
$12 \cdot 100$	$x = 30$	$y = 10\sqrt{3}$
$3 \cdot 4 \cdot 100$		

$$z = 20\sqrt{3}$$



## Example 7

Find the values of  $x$ ,  $y$ , and  $z$ .

<u>leg</u>	<u>leg</u>	<u>alt</u>
$\frac{27+x}{z} = \frac{z}{x}$	$\frac{27+x}{y} = \frac{y}{27}$	$\frac{27}{18} = \frac{18}{x}$
$\frac{39}{z} = \frac{z}{12}$		$\frac{27x}{27} = \frac{324}{27}$
$\sqrt{z^2} = \sqrt{39 \cdot 12}$		$x = 12$

$$z = \sqrt{13 \cdot 3 \cdot 3 \cdot 2 \cdot 2}$$

$$z = 6\sqrt{13}$$

$$\frac{39}{y} = \frac{y}{27}$$

$$\sqrt{y^2} = \sqrt{39 \cdot 27}$$

$$y = \sqrt{13 \cdot 3 \cdot 3 \cdot 3 \cdot 3}$$

$$y = 9\sqrt{13}$$

