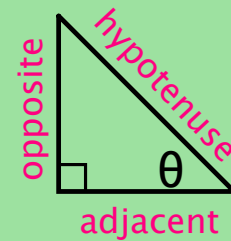


6.2 Part 1 Trigonometry of Right Triangles

Trigonometric Ratio	Abbreviation	Definition
sine of θ	$\sin \theta$	$\frac{\text{opposite}}{\text{hypotenuse}}$
cosine of θ	$\cos \theta$	$\frac{\text{adjacent}}{\text{hypotenuse}}$
tangent of θ	$\tan \theta$	$\frac{\text{opposite}}{\text{adjacent}}$



SOH CAH TOA

$$\text{SOH} = \sin t = \frac{\text{opp}}{\text{hyp}}$$

$$\text{CAH} = \cos t = \frac{\text{adj}}{\text{hyp}}$$

$$\text{TOA} = \tan t = \frac{\text{opp}}{\text{adj}}$$

Other Three Trigonometric Functions

Trigonometric Ratio	Abbreviation	Reciprocal	Definition
cosecant of θ	$\csc \theta$	$\frac{1}{\sin \theta}$	$\frac{\text{hypotenuse}}{\text{opposite}}$
secant of θ	$\sec \theta$	$\frac{1}{\cos \theta}$	$\frac{\text{hypotenuse}}{\text{adjacent}}$
cotangent of θ	$\cot \theta$	$\frac{1}{\tan \theta}$	$\frac{\text{adjacent}}{\text{opposite}}$

Example 1

Find the values of the six trigonometric functions of θ . Give exact answers.

$$\sin \theta = \frac{12}{13}$$

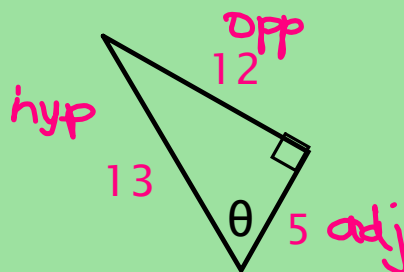
$$\cos \theta = \frac{5}{13}$$

$$\tan \theta = \frac{12}{5}$$

$$\csc \theta = \frac{13}{12}$$

$$\sec \theta = \frac{13}{5}$$

$$\cot \theta = \frac{5}{12}$$

**Example 2**

Find the values of the six trigonometric functions of θ . Give exact answers.

$$\sin \theta = \frac{5}{\sqrt{61}} \cdot \frac{\sqrt{61}}{\sqrt{61}} = \frac{5\sqrt{61}}{61}$$

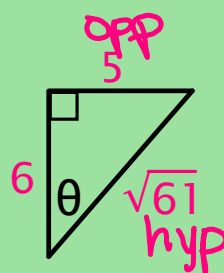
$$\cos \theta = \frac{6}{\sqrt{61}} \cdot \frac{\sqrt{61}}{\sqrt{61}} = \frac{6\sqrt{61}}{61}$$

$$\tan \theta = \frac{5}{6}$$

$$\csc \theta = \frac{\sqrt{61}}{5}$$

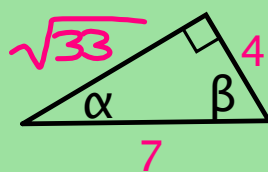
$$\sec \theta = \frac{\sqrt{61}}{6}$$

$$\cot \theta = \frac{6}{5}$$



Example 3

Find (a) $\sin \alpha$ and $\cos \beta$,
 (b) $\tan \alpha$ and $\cot \beta$,
 (c) $\sec \alpha$ and $\csc \beta$.



$$4^2 + b^2 = 7^2$$

$$16 + b^2 = 49$$

$$\begin{array}{r} 16 + b^2 = 49 \\ -16 \quad -16 \\ \hline \sqrt{b^2} = \sqrt{33} \\ b = \sqrt{33} \end{array}$$

$$a) \sin \alpha = \frac{4}{7}$$

$$\cos \beta = \frac{4}{7}$$

$$b) \tan \alpha = \frac{4}{\sqrt{33}} \cdot \frac{\sqrt{33}}{\sqrt{33}} = \frac{4\sqrt{33}}{33}$$

$$\cot \beta = \frac{4}{\sqrt{33}} \rightarrow \frac{4\sqrt{33}}{33}$$

$$c) \sec \alpha = \frac{7}{\sqrt{33}} \cdot \frac{\sqrt{33}}{\sqrt{33}} = \frac{7\sqrt{33}}{33}$$

$$\csc \beta = \frac{7}{\sqrt{33}} \rightarrow \frac{7\sqrt{33}}{33}$$

Example 4 SDH

Find the side labeled x.

$$x \cdot \sin 45^\circ = \frac{12}{x} \cdot x$$

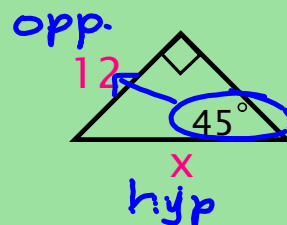
$$\frac{x \cdot \sin 45^\circ}{\sin 45^\circ} = \frac{12}{\sin 45^\circ}$$

$$x = \frac{12}{\frac{\sqrt{2}}{2}}$$

$$x = \frac{12 \cdot 2}{\sqrt{2}}$$

$$x = \frac{24 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}}$$

$$x = \frac{24\sqrt{2}}{2} \rightarrow$$



$$x = 12\sqrt{2}$$

Example 5 TOA

Find the side labeled x.

$$x \cdot \tan 30^\circ = \frac{4}{x} \cdot x$$

$$\frac{x \cdot \cancel{\tan 30^\circ}}{\cancel{\tan 30^\circ}} = \frac{4}{\tan 30^\circ}$$

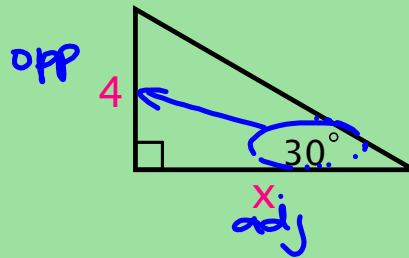
$$x = \frac{4}{\frac{\sqrt{3}}{3}}$$

$$x = 4 \cdot \frac{3}{\sqrt{3}}$$

$$x = \frac{12}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$x = \frac{12\sqrt{3}}{3}$$

$$\boxed{x = 4\sqrt{3}}$$



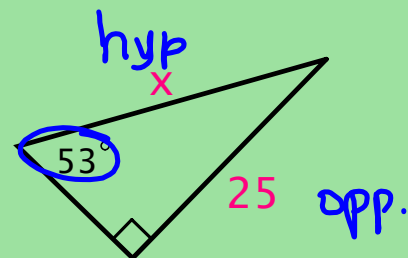
Example 6 SOH

Find the side labeled x. State your answer correct to five decimal places.

$$x \cdot \sin 53^\circ = \frac{25}{x} \cdot x$$

$$\frac{x \cdot \cancel{\sin 53^\circ}}{\cancel{\sin 53^\circ}} = \frac{25}{\sin 53^\circ}$$

$$\boxed{x \approx 31.303}$$

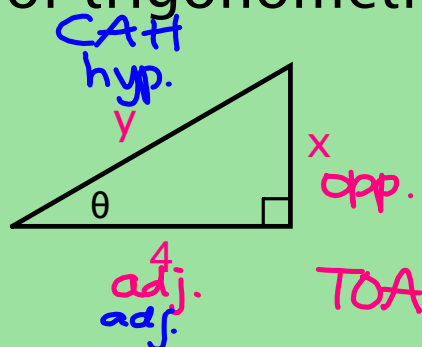


Example 7

Express x and y in terms of trigonometric ratios of θ .

$$4 \cdot \tan \theta = \frac{x}{4} \cdot 4$$

$$\boxed{4 \tan \theta = x}$$



$$y \cdot \cos \theta = \frac{4}{y} \cdot y$$

$$\frac{y \cos \theta}{\cos \theta} = \frac{4}{\cos \theta}$$

$$\boxed{y = \frac{4}{\cos \theta}}$$

Example 8

Sketch a triangle that has acute angle θ , and find the other five trigonometric ratios of θ .

$$\cos \theta = \frac{9 \text{ adj}}{40 \text{ hyp.}}$$

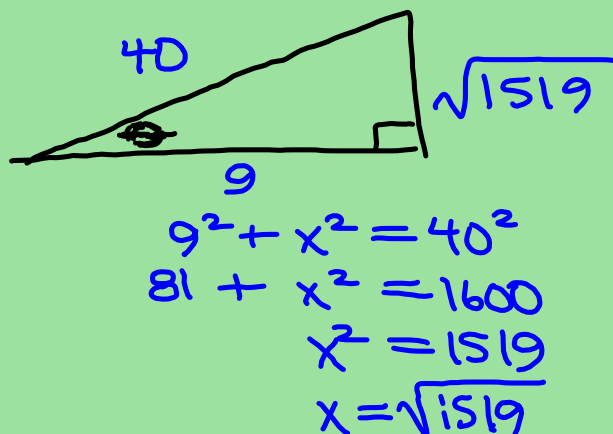
$$\sin \theta = \frac{\sqrt{1519}}{40}$$

$$\tan \theta = \frac{\sqrt{1519}}{9}$$

$$\sec \theta = \frac{40}{9}$$

$$\csc \theta = \frac{40}{\sqrt{1519}} \rightarrow \frac{40\sqrt{1519}}{1519}$$

$$\cot \theta = \frac{9}{\sqrt{1519}} \rightarrow \frac{9\sqrt{1519}}{1519}$$



Example 9

Sketch a triangle that has acute angle θ , and find the other five trigonometric ratios of θ .

$$\tan \theta = \frac{\sqrt{3}}{1} \begin{matrix} \text{opp.} \\ \text{adj.} \end{matrix}$$

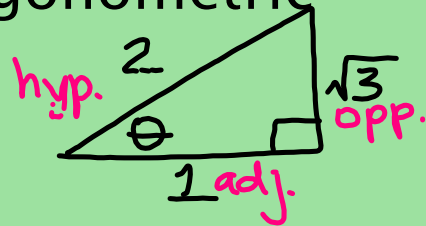
$$\sin \theta = \frac{\sqrt{3}}{2}$$

$$\cos \theta = \frac{1}{2}$$

$$\cot \theta = \frac{1}{\sqrt{3}} \rightarrow \frac{\sqrt{3}}{3}$$

$$\csc \theta = \frac{2\sqrt{3}}{2}$$

$$\sec \theta = 2$$



$$1^2 + (\sqrt{3})^2 = x^2$$

$$1 + 3 = x^2$$

$$4 = x^2$$

$$2 = x$$