Exponential Growth Equation:  \( y = a (1 + r)^t \)

- \( a \) is the initial amount
- \( r \) is the percent increase, written as a decimal
- \( 1 + r \) is the growth factor

EXAMPLE 1:
In 1995, there were 275 cell phone subscribers in Aiken. The number of subscribers increased by 75% per year after 1995. Write an exponential equation that models the number of cell phone subscribers after \( t \) years. How many were there in 2004?

\[
y = a (1 + r)^t
\]
\[
y = 275 (1 + 0.75)^t \rightarrow y = 275(1.75)^t
\]
\[
y = 275(1.75)^9
\]
\[
y \approx 42,333 \text{ subscribers}
\]

EXAMPLE 2:
In the exponential equation \( y = 35 (1.27)^x \), identify:

a) the initial amount, \( 35 \)
b) the growth factor, \( 1.27 \)
c) the percent increase.

\[
1 + r = 1.27
\]
\[
-1
\]
\[
r = 0.27
\]
\[
27\% \text{ increase}
\]
EXAMPLE 3:

Bacteria can multiply at an alarming rate when each bacteria splits into two new cells, thus doubling. If a scientist starts with three bacteria which can double every hour, how many bacteria will she have by the end of the day?

\[ a = 3 \]

\[ \text{doubles} = 100\% + 100\% = 200\% \]

\[ (1 + r) = 2 \]

\[ y = 3(2)^t \]

\[ y = 50,331,648 \text{ bacteria} \]

EXAMPLE 4:

In 1970, the population of a city was about 278,000. Since then, the city population has grown at an average annual rate of 1.8%.

a) Write an exponential equation that models the population of this city \( t \) years after 1970.

\[ y = 278,000(1 + .018)^t \]

b) About how many people lived in the city in 1990? \( t = 20 \)

\[ y = 278,000(1.018)^{20} \]

\[ y \approx 397,192 \text{ people} \]

c) What is the population of this city today? \( t = 42 \)

\[ y = 278,000(1.018)^{42} \]

\[ y \approx 642,969 \text{ people} \]
Exponential Decay Equation: \( y = a (1 - r)^t \)

- \( a \) is the initial amount
- \( r \) is the percent decrease, written as a decimal
- \( 1 - r \) is the decay factor

**EXAMPLE 5:**
Jolene purchases a new car for $22,499. The value of the car decreases by 11% each year. Write the exponential equation that models the car's value after \( t \) years. Then find its value after 3 years.

\[
y = 22,499 (1 - 0.11)^t
\]

\[
y = 22,499 (0.89)^t
\]

\[
y = 22,499 (0.89)^3
\]

\[
y \approx 15,861.10
\]

**EXAMPLE 6:**
In the exponential equation \( y = 200 (0.71)^x \), identify:

a) the initial amount, \( 200 \)
b) the decay factor, \( 0.71 \)
c) the percent decrease.

\[
1 - r = \frac{1}{0.71} = 1.41
\]

\[
-1 \frac{1}{-1} = -1 \frac{1}{-1}
\]

\[
r = 0.29
\]

29% decrease
EXAMPLE 7:
An adult takes 400 mg of ibuprofen. Each hour, the amount of ibuprofen in the person's system decreases by about 29%.

a) Write an exponential equation that models the amount of ibuprofen left in this adult's system after $t$ hours.

$$y = 400(1 - .29)^t$$

$$y = 400(.71)^t$$

b) How much ibuprofen is left after 6 hours?

$$y = 400(.71)^6$$

$$y \approx 51.2 \text{ mg}$$

EXAMPLE 8:
You drink a beverage with 120 mg of caffeine. Each hour, the caffeine in your system decreases by about 12%. Estimate the amount of caffeine in your system after 7 hours.

$$y = 120(1 - .12)^7$$

$$y = 120(.88)^7$$

$$y \approx 49.0 \text{ mg}$$