6.5 Part 2 REMAINDER & FACTOR THEOREMS

REMAINDER THEOREM

If the polynomial P(x) is divided by x - c, then the remainder is the value P(c).

Example 1

Use synthetic division and the Remainder Theorem to evaluate P(c) if $P(x) = x^3 - 2x^2 - 5x + 10$ and $c = 1$.

$$\begin{array}{r|rrrr} 1 & 1 & -2 & -5 & 10 \\ & \downarrow & 1 & -1 & -6 \\ \hline & 1 & -1 & -6 & 4 \end{array} \leftarrow \text{remainder}$$

$$P(1) = \underbrace{(1)^3 - 2(1)^2 - 5(1) + 10}$$

$$P(1) = 4 \leftarrow \text{remainder}$$

Example 2

Find the remainder when $P(x) = 3x^3 + 4x^2 - 2x + 1$ is divided by $x + 3$.

$$x + \cancel{3} = 0$$
$$\quad -\cancel{3} \quad -3$$
$$x = -3$$

$$P(-3) = \underbrace{3(-3)^3 + 4(-3)^2 - 2(-3) + 1}$$

$$P(-3) = -38$$

remainder

FACTOR THEOREM

For a polynomial P(x), x - c is a factor if and only if P(c) = 0.

Example 3

Let $P(x) = x^3 - 7x + 6$. Show that $P(1) = 0$, and use this fact to factor P(x) completely.

$P(1) = (1)^3 - 7(1) + 6$

$P(1) = 0$ ✓

$x = 1$

$-1 \quad -1$

$x - 1 = 0$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -7 & 6 \\ & \downarrow & 1 & 1 & -6 \\ \hline & 1 & 1 & -6 & 0 \end{array}$$

$(x-1)(x^2 + x - 6)$

$(x-1)(x-2)(x+3)$

sum 1 product -6

$\frac{-2}{1} \quad \frac{3}{1}$

Example 4

Let $P(x) = 2x^3 - 3x^2 - 11x + 6$. Show that $P(-2) = 0$, and use this fact to write P(x) in factored form. Then find the zeros.

$x = -2$

$+2 \quad +2$

$x + 2 = 0$

$$\begin{array}{r|rrrr} -2 & 2 & -3 & -11 & 6 \\ & \downarrow & -4 & 14 & -6 \\ \hline & 2 & -7 & 3 & 0 \end{array}$$

$(x+2)(2x^2 - 7x + 3)$

$\underbrace{(x+2)(x-3)(2x-1)}_{\text{factored form}}$

sum -7 product 6

$\frac{-3}{1} \quad \frac{-\cancel{6}}{\cancel{2}} \quad \frac{-1}{2}$

$x + \cancel{2} = 0$, $\cancel{-2} \; -2$, $x = -2$

$x - \cancel{3} = 0$, $\cancel{+3} \; +3$, $x = 3$

$2x - \cancel{1} = 0$, $\cancel{+1} \; +1$, $\frac{2x}{2} = \frac{1}{2}$, $x = \frac{1}{2}$

$\boxed{\text{zeros} = -2, 3, \frac{1}{2}}$

Example 5

Given the polynomial $2x^3 + x^2 - 13x + 6$ and the factor $x + 3$, find the remaining factors and write the polynomial in factored form. Find the zeros.

$x + 3 = 0$, $x = -3$

$$\begin{array}{r|rrrr} -3 & 2 & 1 & -13 & 6 \\ & \downarrow & -6 & 15 & -6 \\ \hline & 2 & -5 & 2 & 0 \end{array}$$

$(x+3)(2x^2 - 5x + 2)$

sum -5 product 4

$\frac{-2}{1}$ $\frac{-4}{2}$ $\frac{-1}{2}$

factored form → $(x+3)(x-2)(2x-1)$

$x+3=0$, $x=-3$ $x-2=0$, $x=2$ $2x-1=0$, $2x=1$, $x=\frac{1}{2}$

$\boxed{\text{zeroes} = -3, 2, \frac{1}{2}}$

Example 6

Given the polynomial $4x^3 + 13x^2 - 37x - 10$ and the factor $x - 2$, find the remaining factors and write the polynomial in factored form. Find the zeros.

$$\begin{array}{r|rrrr} 2 & 4 & 13 & -37 & -10 \\ & \downarrow & 8 & 42 & 10 \\ \hline & 4 & 21 & 5 & 0 \end{array}$$

$x-2=0$, $x=2$ $x+5=0$, $x=-5$ $4x+1=0$, $x=-\frac{1}{4}$

$(x-2)(4x^2+21x+5)$

sum 21 product 20

$\frac{5}{1}$ $\frac{20}{4}$ $\frac{1}{4}$

$(x-2)(x+5)(4x+1)$ ← factored form

$\boxed{\text{zeroes} = 2, -5, -\frac{1}{4}}$